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Issues in the control of stock externality problems with inflexible policy measures

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Issues in the control of stock externality problems with inflexible policy measures

by

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1 PREVIEW

The rapid expansions of economic activities in the recent decades have been accompanied by a number of serious social and economic problems in our society. One of these newly raised economic issues is the increase in non-market interaction among economic agents, which is, so-called, the problem of externality. The rise in non-market interaction, as a consequence of industrialization, is clearly exemplified with the case of the environmental problem.

Clean air or pure water were the typical examples of free goods in the classics of economics.[1] But we frequently observe the cases where a clean environment cannot be obtained without certain cost. Some industries claim their need to emit the discharges into the surroundings to economize on the costs that would otherwise be shifted to the consumers with higher output prices. Residents or firms adjacent to polluters demand their rights to a safe and nondeteriorated environment. Such conflicts between economic agents become more wide-spread and more severe with increased industrialization and more intensive use of the commonly owned resources.

Despite an abundant literature that is already available in the area of externality, some important issues are still unexplored: one of the unanswered questions is the effect of stock externality and government control on this problem. The impor-

[1]For example, A. Marshall (1920, pp. 55-56).

tance of this issue will be apparent with an examination of the previous example.

Some environmental problems, such as air pollution, stream water pollution or noise pollution, are typical examples of flow externality. In these cases, the external effect generated by a polluter can affect the pollutee in the same period and the flow of the discharge is presumed to disappear in the next period. Since any change in the level of emission will be accompanied by an immediate response on the damage side, the policy issues associated with flow externality are relatively simple.

On the other hand, there are certain environmental problems that are arising from the deteriorated state of stocks. Contaminations of groundwater or a lake and pollution in the ocean are typical examples. As compared to flow externality, stock effects are more difficult to handle. A change in the state of a stock takes a long period of time and any remedial policy cannot bring forth an immediate response in curtailing the damage. In fact, our major concerns on environmental conditions arise from the persistent effects from the deteriorated environmental stocks.

This dissertation is written with an object to contribute to the development of the theory on the control of negative stock external effects. The ideal first-best policy is important in the conceptualization of the problem, but its feasibility is not always ensured. This is because an ideal dynamic control requires changing the policy variable at every moment of decision. In the real world, such a continuous change may not be possible and a controlling authority has to rely on inflexible policy measures.

The basic issue of this dissertation is how the policy should be designed for an efficient control of stock externality problem when the policy variable cannot be changed frequently. The models developed in this dissertation are abstracted from the general cases of the industrial pollution problem. But this does not limit the applicability of these models to other cases of stock externality since all of the models are specified in general functional forms.

The plan of this dissertation is as follows. A short review on the theory of externality is given in Chapter 2. For a correct understanding of the basic concept of externality, the review covers the definition of externality, the source of the problem and generally proposed remedies. Based on this review of existing studies in this area, possible directions of future studies have been suggested.

In Chapter 3, Section 3.1 involves an explanation of the source of inflexibility in the control variable and the justification of introducing a fixed one-time control. Section 3.2 examines the characteristics of the first-best policy and its dynamic path. In Section 3.3, the basic properties of the second-best policy are derived mainly by comparing it with the path of the first-best policy.

Chapter 4 considers the issue of the optimal starting point of a fixed one-time control. It is shown that an earlier start of control may not necessarily be desirable in the problem of stock externality if the policy variable cannot be changed freely. Subsequently, a general rule is formulated for the decision of the optimal starting point. Another important result provided in this chapter is that the immediate start of a control is always desirable if the current state of the environmental stock is more deteriorated than its first-best steady state. As a related issue, it is shown that a myopically oriented controller has a strong incentive to defer the start of a control on an externality if the implementation of a control policy requires a great amount of set-up cost.

Chapter 5 deals with the issue of a welfare comparison of two representative control policy measures for the control of a stock externality: a Pigouvian tax system and a quantity restriction. Under a certainty assumption, these two second-best policies are equivalent in terms of economic efficiency. But incorporation of stochastic factors will make one policy different from the other, so the basic question is: under what circumstances can one control mode outperform the other? Comparison of the economic performances of quantity restriction and price control has been an important issue in the literature on the control of externality since Weitzman (1974). The main contribution of this chapter is to extend Weitzman's static framework into a dynamic structure. As a result, it is shown that Weitzman's original model is a particular case of the formulation developed in this chapter.

Chapter 6 presents a short summary on the general contents of this dissertation. Chapter 7 contains several mathematical derivations that cannot be included in the main text, but they are important for the explanation of the results given in the main text.

2 A REVIEW ON THE THEORY OF EXTERNALITY

2.1 Definition of Externality

It may be difficult to avoid a certain degree of subjectivity in any kind of definition, but a clearly defined terminology still seems to be helpful in reducing ambiguity and confusion. A standard way of defining the concept of a certain term is to state what it is. However, the concept of externality[1] is too elusive to define in such a simple way. This explains why the definitions available in the recent literature involve such additional descriptions as when it takes place or what it does.[2] Instead of reviewing the long and tedious arguments concerning the definition of externality, a short summary of the definition that is widely acknowledged among the contemporary economists should be sufficient for the purpose of this dissertation.

A simple example of the relationship between a polluting firm and its surrounding residents might be sufficient in conveying the basic idea of an externality.

[1]Since the definition of pecuniary externality is a controversial issue, the discussion in this section is mainly confined to technological externality. Hence, the term externality without any modification indicates technological externality. As for the definition of pecuniary externality, it will be briefly mentioned at the end of this section.

[2]As a matter of fact, there are quite a few cases, for example Scitovsky (1954), Buchanan and Stubblebine (1962), and Meade (1952 and 1973), in which the definitions have been provided in the standard way of statement. But most of them were easily exposed to criticism.

The implication of this example is that the surrounding residents' utility functions involve the level of emission as their arguments. An important aspect of the situation is that the pollutees do not have any choice in the level of pollution. Instead, they are forced to "consume" the given amount of pollution that is determined by the polluter's unilateral decision. By generalizing this example, we can say that an externality is present whenever one agent's utility or production function involves a real variable that can be controlled, not by himself, but by (the economic activity of) other(s).

It is important to note that the concept of externality basically relies on the system of market economy. The first fundamental theorem of welfare economics indicates that, under ideal conditions, a market economy automatically results in a position of Pareto optimality, provided that individuals maximize their utilities and firms maximize their profits. Such a market performance is based on the fact that each individual is not exposed to the direct influence of another's behavior as in the case of pollution. Under the above mentioned ideal conditions, the information on other peoples' decisions is transmitted through the price in a market and each individual can freely decide his own production or consumption activity based on the market information. When there exists an externality, one agent's behavior directly influences the other's welfare state without any consent from the affected party and market economy does not lead to Pareto optimality unless a remedial policy is exercised. In other words, externality is one form of market failure.[3]

When we indicate a particular economic incident to be a market failure, what

[3]A good summary on the conditions of market failure is provided in the classics of Bator (1958).

we basically presume is an economic system where successful performance of the market is a general phenomenon. Hence, the existence of an externality is an exceptional case in this economy. This implies that the meaning of externality can be found in the context of a market economy. The pollution example given above can be observed in any type of economic system, but it may not make much sense to call it an externality when it takes place in an economy that is not based on a market system.

Even though the preceding statements provides the basic concept of externality, several qualifications are required to complete the definition. First, an externality presupposes that the affected party does not receive (or pay) proper compensation for the damage (or benefit) from the party who generates the external effect. Second, either the damage or the benefit from an externality should not be deliberate but accidental. These are important aspects of externality, so we need to examine these qualifications more closely.

The first statement that the party affected from an externality does not receive (or pay) any compensation is directly related to the perception of externality as an absence of market. As has been suggested by Arrow (1970), the externality-generating factor, which is corresponding to the level of emission in the previously given example, can be interpreted as an additional good. Since the production or the allocation of this good (or bad) is not determined by the mechanism of a market system, the economy might not be in an optimal state. However, a possible supposition is that if there had existed a market for this good (or bad), then such an inefficiency could have been avoided. Consequently, we can attribute the source of market failure not to the simple existence of such an uncontrollable variable in an agent's utility or production function but to the absence of a market that can guarantee the transaction of such an externality generating good.[4] Therefore, the identification of the source of externality can be concentrated on the examination of why the market has not been developed. This issue will be reviewed more specifically in the subsequent section of this chapter.

The other condition that an externality is not a deliberate but an accidental result was indicated by Mishan (1971). Baumol and Oates (1988) explained it with an interesting example. "If I purposely maneuver my car to splatter mud on a pedestrian whom I happen to dislike, he is given no choice in the amount of mud he 'consumes', but one would not normally regard this as an externality."[5] The mud-splattering action can be explained with the assumption that the pedestrian's unhappiness is the source of my pleasure. In the context of an economic model, the same explanation can be made in such a way that the degree of pedestrian's indignation (or indirectly, the amount of mud inflicted on the pedestrian) is the explanatory variable of my utility function. The concept of externality excludes such a feedback effect on the production or consumption function of the agent whose activity generates spill-over. Therefore, an externality should be a simple by-product or side-effect of a certain activity. Defining externality as an accidental result is important because such a view determines the corrective policies on externality.

There is another subsidiary question in relation to the use of the terminology. In the case of a negative externality, social optimality does not require a complete elimination of the external effect because a reduction in the externality is accom-

[4]Heller and Starrett (1976) and Cornes and Sandler (1986) are on the same line of interpretation as that of Arrow (1970).

[5]Baumol and Oates (1988, p. 17).

panied by the contraction of the externality-causing activity which is presumably desired by the society. The question is whether we can say that the externality has been completely removed if the exercise of a proper policy measure has changed the state of the economy into a social optimality. For such a question, "it seems more natural to say that the externality has been reduced to an appropriate level, rather than it has been eliminated altogether."[6] Therefore, a common usage of the word, externality, is to indicate a potential state of market failure when there is no correction on it, not the actual state that may have been properly corrected.

So far, our attention has been paid to the definition of technological externality which is the main issue in the current literature on externality. In the past, some economists would have given more emphasis to the notion of pecuniary externality than that of technological externality.[7] But such a view is not true today. In addition, the definition of pecuniary externality is somewhat subjective and arbitrary.

For instance, Scitovsky (1954), and Heller and Starrett (1976) limited the concept of pecuniary externality to the case of market failure that can be shown in such an example as rail and steel industries. When the steel industry exhibits economies of scale, even if neither industry finds it profitable to go it alone, there is still the possibility of the co-existence of the two industries. The demand from the rail industry can expand the total demand of steel up to a sufficient level at which the operation of the steel industry becomes profitable. Under such a condition, the competitive market system cannot lead to the co-existence of the two industries

[6]Baumol and Oates (1988, p. 18).

[7]Scitovsky (1954), for example, argued that technological externalities are rarely observed in the real world and unimportant while pecuniary externalities are prominent forms of externality, particularly in the developing countries, and that economists should pay more attention on the latter concept of externality.

unless perfect futures market is available. Therefore, it is a state of market failure and it has sufficient common characteristics with technological externality.

On the other hand, Mishan (1971) and Baumol and Oates (1988) dismissed such a complicated concept. They defined a pecuniary externality to be a case in which one individual's activity affects the welfare condition of the other through a market. For example, an increase in the demand of the shoe industry raises the price of leather, thus affecting the welfare of hand bag consumers. Therefore, it does not involve any concept of market failure when the interaction occurs in a set of decentralized markets. This definition seems to have been derived directly from the concept of external economies in Viner (1953). Since the definition is confusing and the concept is relatively unimportant, pecuniary externality will not be mentioned in the subsequent part of this dissertation.[8]

2.2 Source of Externality

Once the existence of externality is known to be a major source of market failure, our next question is how to correct this problem. But it is important to identify the source of the problem as a basis for the direction of remedial policies. As indicated earlier, an externality can be identified to be the absence of a market. Such an interpretation allows us to point out the sources of externality with less difficulty. What we need is to show the causes of why the market has not been developed. Several factors can be listed as the potential sources of the non-existence of a market: difficulties in defining property rights, market operating costs, and limited number

[8]Mishan (1971), for example, denied usefulness of the concept of pecuniary externality as "superfluous and possibly confusing" (p. 8).

of buyers and sellers.[9]

Since a market transaction consists of the voluntary transfer of the property rights, the development of a market automatically requires that the property right over the commodity be easily defined and enforced. In most cases of externalities, it is difficult to define the property rights because of technical factors or social customs. In general, externalities have inherent public characteristics,[10] since the exclusion (of the benefit) or avoidance (of the damage) cannot be accomplished with an affordable level of cost. In the case of a pollution problem, it is difficult to define property rights over water or air because of the elusive and migratory characteristic of these bodies. This is the main reason why a voluntary market for pollution cannot be developed.

There are other cases in which the property rights have been clearly defined but high enforcement costs do not allow the actual protection of the exclusive rights over them. For example, authorships and patents are entitled with well-defined property rights but they are infringed on due to high enforcement costs. Aside from the issue of legality, such a condition easily leads to the problem of free riding or easy riding[11].

Operation of a market necessarily requires a certain amount of cost. When the cost involved in the operation of a market exceeds the resulting benefit, there

[9]This classification is according to Cornes and Sandler (1986).

[10]Some externalities have a private nature, which Baumol and Oates (1988) defined as exhaustible externality. An example is the refundable cans thrown away in a public place. These cans generate both positive externality (as the potential source of income for the collectors) and negative externality (as the source of public nuisance). The negative externality, in this case, has a public nature, while the positive externality has a private nature.

[11]Terminology due to Cornes and Sandler (1984).

would not be any incentive to develop a market voluntarily. In certain instances, direct regulation by government has a cost-saving effect as compared to a voluntary agreement between the two directly related parties.

The small number of buyers and sellers has a certain relationship with the market operating cost. *Ceteris paribus*, the smaller the number of participants in the market, the higher per unit transaction cost, if the portion of fixed cost is high. In addition, a limited number of buyers and sellers may show non-competitive behaviors. Due to the individual buyers' and sellers' abilities to influence the market, the transactors may not necessarily reveal the correct information and the market could easily break down.

2.3 Remedies of Externality

2.3.1 Decentralized market

Even though there are some obstacles in developing voluntary markets, these difficulties may not necessarily justify direct interventions of a government. A market has certain type of public characteristics. Since a market continues to operate for a sufficiently long period, the organization of a market and the information available from it can be employed not only by the current participants but also by those who come later. Potential participants who will use the market only for limited periods would appreciate the value of the market much less than its real social value. Because of the divergence between the private value and the social value of a market, there are certain cases where a group of potential participants for an undeveloped market may not take up the burden of high set-up costs. In other words, an intertemporal externality prevails in the establishment of a market for externality. If a great amount of set-up cost is involved in the initial organization of a market, the sufficient role of the government is only to remove the divergence between the private benefit and the social benefit of the market.

2.3.2 Direct negotiation

A decentralized market is not the only condition for a voluntary transaction of an externality. As Coase (1960) argued, people can directly negotiate for the improvement of mutual benefits in the case of small-numbered participants. The implication of this argument is broad and diverse. It simply requires negotiability as the condition for an economic efficiency and looks more robust and general than the welfare theorem that is based on the condition of universality of decentralized markets. As an immediate result, the role of government, under such an argument, is not to intervene directly but to provide an indirect assistance to remove the barriers that work against direct negotiation. Defining the property right is the first step for a voluntary negotiation. Another point of the Coase theorem indicates that Pareto optimality can be attained regardless of the assignment of property rights between polluter and pollutee.[12] This fact can reduce the burden of a government which has to find the best way to the division of the property rights between the two parties related with the externality.

On the other hand, a possible criticism of the Coase theorem is that "it is a tautology that if people negotiate efficiently then every outcome will be efficient

[12]But the optimal level of externality may not always be equal under different type of property right assignment because of income effects.

(else people would negotiate something better)."[13] In another criticism, it was argued that the core in the Coase theorem is empty.[14] In fact, the importance of voluntary agreement is found more in a theoretical conceptualization than in a practical application. In the majority of cases, there are big barriers to defining property rights on common property not only because of technical factors, but also because of social or customary conditions. Moreover, the associated transaction and enforcement costs may be too high to reach any voluntary agreement.

2.3.3 Tax and subsidy

The most frequently mentioned policy measures aiming at externalities are Pigouvian taxes and subsidies, which are used to control the divergence between marginal private cost and marginal social cost. A tax (or a subsidy) in this context is an additional cost (or reward) to a certain economic activity. However, there arises asymmetry between the price paid by one party and the price received by the other party. Under a negative externality, the pollutee receives zero compensation but the polluter pays a positive amount of tax; under a positive externality, the beneficiary pays zero price and the benefactor receives a positive sum of subsidy.

A possible interpretation of the asymmetric prices is that the initial divergence of marginal social cost and marginal private cost has been substituted by another divergence of the two prices perceived by the polluter (benefactor) and the pollutee (beneficiary), respectively. Accordingly, some doubt has been raised whether asymmetric pseudo-prices under a Pigouvian tax system or a subsidy scheme can bring

[13]Farrell (1987, p. 113).

[14]Aivazian and Callen (1981) and the response of Coase (1981).

the economy to an optimal state. The question has become more complicated with the argument that the victims also need to be taxed. This view is based on the reciprocal nature of externality: without the existence of the pollutee, no externality is possible, so the pollutee is also responsible for the problem arising out of externality. Concerning the question, whether the victim of a negative externality should be taxed or subsidized or neither,[15] the general conclusion is that, in a competitive market, only the tax on the polluter would be sufficient for optimality.[16]

The limitations of a Pigouvian tax or a subsidy in a second-best setting have been mentioned in several papers. A Pigouvian tax or a subsidy can provide a correct signal to the production or consumption of the externality generator only when the market condition is in a decentralized state. Davis and Whinston (1962) showed the problems of a Pigouvian tax in an oligopoly market. Buchanan (1969) indicated that a monopoly polluter might actually underpollute as compared to the socially optimal level and need to be subsidized, instead of being taxed. In addition to such limitations, Carlton and Loury (1980) proved the fact that a Pigouvian tax may not be efficient in the long run.

The issue of fairness might be one of the most important factors that prevent Pigouvian tax system from being adopted for the solution of a real world problem.[17] The correct rate of a Pigouvian tax is determined not by the rate of emission but

[15] As for this issue, the representative contributions are Coase (1960), Turvey (1963), Olson and Zeckhauser (1970), Baumol (1972), and Baumol and Oates (1988).

[16] This conclusion is based on the following reasons. A compensation to the victims may reduce their incentives to protect themselves from the pollution, which is the case of moral hazard. A tax on the victims is also redundant since the level of pollution will keep the pollutees away.

[17] Faulhaber and Baumol (1988, p. 582).

by the resulting damage on the pollutees. Consequently, substantial differences in the tax rate are inevitable for the same level of emission. Such a discriminatory variation in the tax rate may not be adopted without much friction with the existing social convention.

There are two different types of subsidies as corrective measures of externalities. One is the reward for the benefactors of a positive externality (the subsidy mentioned in the preceding part indicates this kind of subsidy); the other is the subsidy to be paid to the generators of a negative externality in proportion to the reductions in their emission. The latter one is actually an alternative to a Pigouvian tax and both are equivalent under certain situations. Identification of the conditions for an equivalence of the subsidy and a Pigouvian tax has been an important issue in a number of contributions.[18]

2.3.4 Merger and direct regulation

One possible way to internalize the externality is to merge the polluter(s) and the pollutee(s) into one decision unit. However, this remedy can only be applicable to the case where the related parties are firms and their number is limited. It is hard to imagine merging two (or several) households or combining firms and households into the same decision unit.

There are several types of direct regulation: direct quantity restrictions, indirect regulation through the control of the commodities which are related with an externality, or zoning certain activities. A quantity restriction (or quota) is equiv-

[18]The issue has been dealt with in Kamien, Schwartz, and Dolbear (1966), Bramhall and Mills (1966), Tullock (1966), Plott and Mestelman (1968), Porter (1974), Dewees and Sims (1976), Just and Zilberman (1978), Sims (1981).

alent to a Pigouvian tax under the assumption of certainty,[19] but the introduction of uncertainty makes a distinction of one policy from the other. This issue will be examined further in Chapter 5. Indirect regulation on the production or consumption of related good is a possible second-best policy when the first-best policy is not available for such reasons as prohibitively high cost or institutional/customary barriers to a direct regulation.[20]

As a matter of fact, such direct controls as an outright prohibition, zoning, or strict regulation in quantity can be observed more frequently than ideal policies such as Coasian type negotiation or Pigouvian tax or subsidy. Direct regulations are important control measures as far as administrative convenience is concerned and the effect is relatively clear and immediate. However, from the purely economic point of view, these are often naive and inefficient.

2.4 Issues for Further Studies

These days, the study of externalities is one of the most important areas in welfare economics. The issues are numerous and the publications are myriad. An extensive review on the theory of externality is beyond the scope of this dissertation. But a short review provided in the previous sections of this chapter can illuminate, to a certain extent, the general direction taken by the existing studies. This is important as a guide for future studies.

At this stage, several points can be indicated as deficiencies in the existing literature of externality. One of the greatest limitations is that the externality problem

[19]This result is similar to the equivalence theorem between tariff and quota in international economics.

[20]Koenig (1985), and Lapan and Choi (1987) considered this issue.

is generally analyzed in a static framework. This fact is clearly shown in the definition, causes, and remedies of externality. Static analysis is the standard technique of economics. It is simple and manageable. In the specific case of environmental issues, some problems, such as pollution in the air or in stream water, can be described in the context of a flow concept. But a more careful examination of real world problems indicates that our major concern lies in the persistent externalities from deteriorated environmental stock. The externalities arising from stocks, sometimes coupled with the characteristics of irreversibility, become more critical problems.

In a stock externalities, today's contamination affects the production or consumption activities tomorrow. The time lag between the cause and the effect is sometimes greater than a generation. This is completely different from atemporal market failure and cannot be analyzed with the traditional concept of static externality. Consequently, the remedies based on conventional framework cannot yield the intended result. As for such practical needs, economists have not yet answered sufficiently. The literature on dynamic analyses of intertemporal spill-over is relatively scarce.[21] Moreover, the limitation of economics is obvious in the issue of intergenerational spill-dovers; what makes the problem more complicated is that

[21] In such areas as fishing or soil erosion, dynamic models are frequently adopted. But the externalities associated with such issues are somewhat different from the general externality case because the majority of the models in these areas are specified in such a way that the externalities among individual agents are internalized within the same industry. An exceptional case is Shortle and Miranowski (1987). More general externality models are available in the issue of growth and pollution in Forster (1973 and 1977), Cropper (1976) and in the dynamic anomalous case of Brito (1972) and Brito and Intriligator (1987).

ethical issues are intermingled with economic efficiency problems.[22] In spite of these difficulties, the actual demand of the real world requires that the stock externality problem be a very important issue in the future studies of this area.

The other shortcoming in the existing literature on externality is that most of the analyses have been conducted in the framework of partial equilibrium. According to the general theory of second best, there is no guarantee that a piecemeal policy leads to a welfare improvement. One known fact is that if one sector shows a greater percentage of divergence of the (implicit) market price from its social cost than the other sectors of the economy, then the correction policy of equating social marginal cost and market price of this particular sector is likely to raise the overall welfare of the economy.[23] This is a useful information but may not be sufficient justification to use a partial equilibrium analysis in every case of externality. The development of general equilibrium analyses would clarify many unidentified and unanswered questions.

There are several other areas that can be indicated as deficient in the existing studies of externality. For instance, regional variation and mechanism design to reveal correct information are also important issues. A negative externality can affect the others only when they stay within certain boundaries of influence. The location of each pollutee and the number of these pollutees determine the level of social cost of the pollution. The locational factor is an important explanatory variable of the damage from pollution but it is very difficult to incorporate it in the representation of production or utility functions, which have generally been

[22]For example, see Dasgupta and Heal (1979, Chapter 10), d'Arge, Schulze, and Brookshire (1982) and Schulze, Brookshire and Sandler (1981).

[23]Mishan (1971).

developed without such arguments as time or location.

In the absence of a decentralized market, the scarcity of information raises a much more difficult problem. Agents are not likely to provide correct information at the request of the regulator. The issue, therefore, is how to design an incentive scheme that makes the individual agent reveal the correct information. This is an important issue with a point source pollution where the source of pollution is identified.[24] But the problem of information is even more severe in the control of non-point source pollution.

Non-point source pollution implies that sources are diffuse or occur over a substantial area in response to some land activity; hence, pollutants are not traceable to discrete sources. Therefore, the lack of information is apparently implied by its definition. The design of an incentive scheme to reveal correct information, which is the basis for a relevant policy, is a challenging issue in the control of non-point source pollution.[25]

[24] Existing studies on incentive schemes to reveal correct information are Kwerel (1977), Dasgupta, Hammond, Maskin (1980), and Repullo (1982). All of them are about point source pollution.

[25] A recent paper on non-point source pollution and mechanism design is Segerson (1988). The issue in this paper is the problems associated with lack of specific information on the level of emission from individual sources who are collectively identified to be polluting.

3 BASIC PROPERTIES OF THE OPTIMAL CONTROL AND ONE-TIME CONTROL POLICIES

3.1 Introduction

In general, negative externality problems arising from certain forms of stock are more difficult to handle than the externalities associated with flows. Once the state of a stock has deteriorated, the negative effect will persist for a prolonged period. Since the problem is the accumulated result of the past history, the remedy may not be in effect in a short period of time and that is why intertemporal considerations cannot be ignored in a stock externality problem.

In spite of the potential seriousness of this problem, the policy measures aimed at the stock externality have not been studied as much as those of its flow counterparts. It is mainly due to the fact that stock effect can only be analyzed in a dynamic context. In a dynamic control of an economic problem, an analytical model is necessary for conceptualization and understanding of the basic characteristics. But there is relatively small room for the analytical results to be directly applied to the formulation of a feasible economic policy. Following an optimal path is a very complicated business and it may be neither desirable nor feasible for a number of reasons.[1]

[1]For example, lack of information, high adjustment costs, or institutional

Except in the special case of a steady state, a dynamic optimal path needs to change the policy variable at every moment of decision. However, such a continuous exogenous change in the policy variables, such as the rates of emission charges or the emission quantity constraints, may not be feasible in the real world. Probably, this is one of the most important factors that keep a theory in stock external problems from being utilized in the formulation of a practical policy.

In a market economy, imposition of a tax implies exercising administrative power over individuals' property rights, which, in turn, requires a legal basis. In most cases, a legislative procedure takes a long time and the decision may be controversial. This means that a change in a policy variable will be associated with very high administrative and adjustment costs as well as the frictions between the parties involved in the externalities. In the case of other policy measures such as quantity restrictions, the same sort of costs and difficulties will be involved in the frequent change of the restricted quantity. Based on realistic institutional constraints, one may naturally ask the following questions: when a control variable cannot be changed for a long period of time, how will the economic decisions and results associated with these fixed policy measures differ from those of the continuous optimal control? The purpose of this chapter is to answer these questions by analyzing the characteristics of the optimal control and those of a one-time control policies in a simple framework.

In Section 3.2, the basic assumptions upon which all the subsequent analyses will be based are described, and then the dynamic path of the first-best policy is shown. In Section 3.3, several basic properties associated with one-time fixed control

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difficulties.

are derived mostly by comparing the levels of the control or the state variables with their counterparts in the first-best policy.

3.2 Properties of the First-Best Solution

In order to exhibit the characteristics of the problem and to raise basic policy issues associated with a stock externality problem, a very simple dynamic model is given in this section. Equation (3.1) is the law that governs the change of the state of an externality-generating stock.

$$\dot{q}_t = \alpha X_t - \delta q_t, \tag{3.1}$$

where q_t represents the amount of an externality-generating stock at time t, such as the level of pollutants in the air of a certain region or the contaminants in the body of a groundwater or a lake. Therefore, a higher level of q_t represents a greater deterioration of the environment. The other variable X_t represents the rate of emission. The equation of motion embodies the assumption that a certain portion (α) of the emission is merged into the accumulation of the contaminant at the same moment when it is discharged and the rest is dispersed. On the other hand, the contaminant is to be depreciated at the rate of δ due to dispersion, decay, or some other factors of natural amelioration.

From the standpoint of a polluter,[2] pollution is a by-product associated with a certain production (or consumption) behavior. When the emission is lessened to a level lower than that of the no-control state, this constraint will reduce the polluter's

[2]In what follows, a single polluter is implicitly assumed, but this assumption will not limit the argument's generality.

benefit, presumably at an increasing rate. If this is an acceptable assumption, then a concave benefit function can be specified as $B(X_t)$ where $B_1(X_t) > 0$ and $B_{11}(X_t) < 0$.[3] This means that, in the relevant domain of X_t,[4] a polluter can have higher benefit when he is induced to (under tax system) or allowed to (under quantity restriction) emit a greater amount of discharge and the marginal private benefit of X_t is diminishing.

On the damage side, the damage function will be specified in a simple form such that it has only one argument, q_t, like $D(q_t)$. It would be admissible to assume that as the level of contaminant (q_t) rises up, the associated damage will increase at an increasing rate, which means the damage function is convex in q_t, *i.e.*, $D_1(q_t) > 0$ and $D_{11}(q_t) > 0$. On the condition that both of the functions $B(\cdot)$ and $D(\cdot)$ are expressed in the same unit,[5] the objective function can be represented in the following form:

$$V(q_0) = \max_{\{X_t\}} \int_0^T e^{-rt}[B(X_t) - D(q_t)]dt + e^{-rt}V(q_T), \tag{3.2}$$

$$\text{subject to } \dot{q}_t = \alpha X_t - \delta q_t,$$

[3]The subscript of a function will represent the derivative of the function. If the function has more than one argument, subscript i represents the derivative with respect to its i*th* argument.

[4]There must be an upper limit in X_t. Suppose that a polluter emits $\bar{X}$ when there is no control on the emission, then the relevant domain of X_t is the closed interval of $[0, \bar{X}]$.

[5]It is assumed that the common unit is a money term, which can allow us to interpret the marginal value of the social external cost of X_t to be the optimal Pigouvian tax rate.

where r represents a social rate of discount. The objective of the controlling authority is to maximize all the stream of discounted future net benefit from time 0 to the terminal time T as well as the discount social value of the terminal state, *i.e.*, $e^{-rT}V(q_T)$.

The current-value Hamiltonian associated with Equation (3.2) is:

$$H_t = B(X_t) - D(q_t) + \lambda_t[\alpha X_t - \delta q_t]. \tag{3.3}$$

First-order conditions are:

$$\frac{\partial H_t}{\partial X_t} = B_1(X_t) + \alpha\lambda_t = 0, \tag{3.4}$$

$$\frac{\partial H_t}{\partial q_t} = -(\dot{\lambda}_t - r\lambda_t) = -D_1(q_t) - \delta\lambda_t, \tag{3.5}$$

$$\frac{\partial H_t}{\partial \lambda_t} = \dot{q}_t = \alpha X_t - \delta q_t. \tag{3.6}$$

If we assume a finite time horizon, the transversality condition is:

$$\lambda_T = V_{qT}. \tag{3.7}$$

However, the problem can be explained better on an infinite time horizon because we cannot conceptualize a specific terminal date T, and/or an exogenous terminal condition $V(q_T)$. Using an infinite time horizon, the transversality condition ceases to be valid, so the assumption of convergence to the steady state is substituted for the transversality condition on the presumption that the environment is stationary.[6] The objective function on an infinite time horizon is restated as:

$$V(q_0) = \max_{\{X_t\}} \int_0^\infty e^{-rt}[B(X_t) - D(q_t)]dt + e^{-rt}V(q_T), \tag{3.8}$$

$$\text{subject to } \dot{q}_t = \alpha X_t - \delta q_t,$$

By assuming that the initial state of the environment q_0 is sufficiently close to the potential steady state level q_s^*, the behavior of the system can be explained by analyzing the movement of the variables around the steady state. Equations (3.5) and (3.6) can be rearranged in the following forms:

$$\dot{\lambda}_t = (r + \delta)\lambda_t + D_1(q_t), \tag{3.9}$$

$$\dot{q}_t = \alpha X(\lambda_t) - \delta q_t. \tag{3.10}$$

Since the two forcing functions $D_1(q_t)$ and $\alpha X(\lambda_t)$ are non-linear functions of the state variable q_t and the costate variable λ_t respectively, the set of differential equations (3.9) and (3.10) can be made tractable with linearizations. By taking Taylor expansions around the steady state point (q_s^*, λ_s) and retaining only the linear terms, the simultaneous differential equations around the steady state point can be approximated in the following linear form.[7]

[6] For a specific explanation about a control problem on an infinite time horizon, refer to Kamien and Schwartz (1981, p. 159).

[7] The additional subscript s represents the value of the function evaluated at the

$$\begin{bmatrix} \dot{\lambda}_t \\ \dot{q}_t \end{bmatrix} = \begin{bmatrix} r+\delta & D_{11s} \\ \alpha X_{\lambda s} & -\delta \end{bmatrix} \begin{bmatrix} \lambda_t - \lambda_s \\ q_t - q_s^* \end{bmatrix}, \tag{3.11}$$

where $X_{\lambda s}$ is positive.[8] The characteristic equation associated with the above differential equation system is:

$$k^2 - rk - \delta^2 - r\delta - \alpha X_{\lambda s} D_{11s} = 0. \tag{3.12}$$

The eigenvalues of equation (3.12) are:

$$k_1, k_2 = \frac{r \pm \sqrt{(r+2\delta)^2 + 4\alpha X_{\lambda s} D_{11s}}}{2}. \tag{3.13}$$

Since both terms inside the bracket are positive, the roots are real and the eigenvalues are real and of opposite sign. Consequently, the stationary point is a saddlepoint as shown in Figure 3.1, which is drawn in the fourth quadrant because q_t takes a positive value while λt takes a negative one.[9]

steady state. The asterisk (*) represents the first-best solution; but the asterisk on the costate variable λ_t is suppressed because it always represents the first-best solution in this dissertation and there is not any possibility of confusion.

[8]From equation (3.4), the following relationship can be derived:

$$X_{\lambda t} = \frac{dX_t}{d\lambda_t} = -\frac{\alpha}{B_{11}} > 0.$$

[9]Since λ_t is the social value of a unit of negative externality generating stock, it will take a negative number, which can be shown with the solution of equation (3.9), *i.e.*,

$$\int_t^\infty e^{-(r+\delta)(\tau-t)} D_1(q_\tau) d\tau < 0, \text{ since } D_1(q_\tau) > 0.$$

The optimal path is the arrowed curve denoted as *aa*. Suppose that the current state is at q_0. Then, in order to approach the steady state point which is the ultimate destination of the movement, we need to assign λ_0 of value to the costate variable at the beginning of the control and continuously change the value of the costate variable while moving along the saddle path. Equation (3.4) tells us that assigning a certain negative value to the costate variable has the economic implication of imposing a tax on X_t. If there is no control on the emission, the polluter's profit (or utility) maximization condition is $B_1(X_t) = 0$. An imposition of tax at the rate of p_t on the emission will change the profit (or utility) maximizing first-order condition into $B_1(X_t) = -p_t$. So an assignment of a certain negative value (λ_t) to the costate variable will leads to the equivalent result of imposing a tax at the rate of p_t per unit of discharge, where $p_t = -\alpha\lambda_t$. Quantity restriction will have the same consequence of economic efficiency as that of a tax imposition so long as the emission amount in each period is controlled to be the same as that under a tax.

If the initial value q_0 is lower (higher) than q_s^*, the first-best optimal tax rate starts at the level lower (higher) than the steady state tax rate $-\alpha\lambda_s$ and approaches $-\alpha\lambda_s$ asymptotically from below. Since the polluter's decision is made according to the first-order condition that $B_1(X_t) = p_t = -\alpha\lambda_t$ and the tax rate and the emission level are in such a negative relationship that $dX_t/dp = 1/B_{11}(X_t) < 0$, the emission level X_t will change to the opposite direction of the change of the absolute value of λ_t. As shown in Figure 3.1, if the initial level of q_0 is lower than q_s^*, then an increase in the absolute value of the costate variable will be associated with a gradual reduction of the emission level from X_0^* to X_s^*, where X_0^* and X_s^* satisfy the condition that $B_1(X_0^*) = -\alpha\lambda_0$ and $B_1(X_s^*) = -\alpha\lambda_s$.

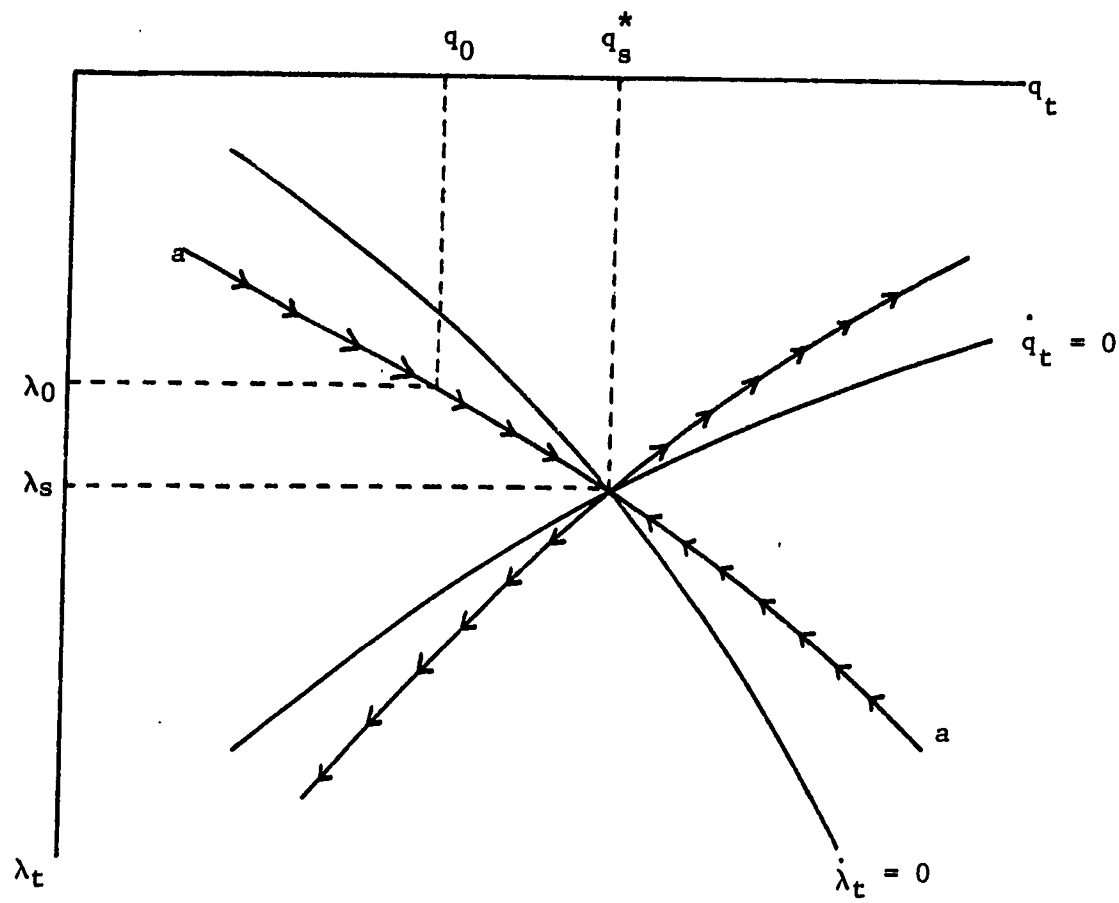

Figure 3.1: Dynamic path of the first-best policy

All the preceding findings in this section can be summarized in the following proposition.

Proposition 1 *If the initial state of the environment is less contaminated than the optimal steady state* ($q_0 < q_s^*$)*, then the optimal emission level at the beginning of the control is greater than the steady state level of emission* ($X_0^* > X_s^*$)*, and the optimal level of emission keeps on decreasing over time* ($\dot{X}_t^* < 0$)*, but the state of the environmental condition will continuously be deteriorating over time* ($\dot{q}_t^* > 0$)*, approaching the steady state level* q_s^* *asymptotically; if* $q_0 > q_s^*$*, then all the reverse will hold, such that* $X_0^* < X_s^*$*,* $\dot{X}_t^* > 0$*, and* $\dot{q}_t^* < 0$.

3.3 Properties of One-Time Control Policy

In the preceding section of this chapter, several reasons have been listed as to why it is difficult to change the policy variable of an exogenous control on an externality problem. Some of these constraints are incurred from economic reasons and others arise from non-economic sources. The main point of this section is to clarify the basic characteristics of a one-time control second-best policy by comparing its path with that of the first-best one. By one-time control, we mean that the control variable is to be determined at one moment of time and to be kept at the same level for the entire planning horizon without any change.

One can, however, easily raise doubts about the validity of assuming a completely rigid control variable that is supposed to be unchanged throughout all the subsequent periods. The reality seems to be that a fixed level of control remains to be effective only for a finite time span until the time of the next amendment or during the period of prescription. To a large extent, such a divergence of the assumption from the reality can be overcome with a discounting factor. The positive rate of a discounting factor will give a greater weight to the effect in the current period or those in the near future at the expense of those in the distant future. There will not be any great difference in the controller's current decision making whether a policy can be changed far in the future or completely fixed forever. On this observation, we can expect that the assumption of a perfectly rigid control variable can approximate the problems of institutional restrictions in the real world where the policy variables are fixed for a long but only finite period of time. The problem arising from a divergence of the assumption from reality becomes smaller as the rate of discount increases.

In a deterministic setting, a constant level of tax rate will fix the rate of emission at a certain level just as in the case of a fixed quantity restriction. If the fixed level of emission is X, then the change of q_t is expressed in a linear first-order differential equation with a constant term, such that $\dot{q}_t = \alpha X - \delta q_t$. Consequently, the state of an externality generating stock at time t is representable in a simple form as follows:

$$q_t = (q_0 - \frac{\alpha X}{\delta})e^{-\delta t} + \frac{\alpha X}{\delta}, \tag{3.14}$$

where q_0 is the state of the stock at the initial time of control.

For a controller who tries to maximize the whole future flow of discounted net social benefit with a tax which begins to be imposed on the emission at $t = 0$, the objective function is

$$J_P(q_0) = \max_{p} \int_0^\infty e^{-rt}[B(X(p)) - D(q_t)]dt + e^{-rt}V(q_T), \qquad (3.15)$$

$$\text{subject to } \dot{q}_t = \alpha X(p) - \delta q_t,$$

where p represents the fixed rate of Pigouvian tax imposed on the emission. The level of emission will have a negative relationship with the tax rate p. The first-order condition associated with the above objective function is:

$$\int_0^\infty e^{-rt}\left[\frac{dB(X)}{dX}\frac{dX}{dp} - \frac{dD(q_t)}{dq_t}\frac{\partial q_t}{\partial X}\frac{dX}{dp}\right] = 0. \qquad (3.16)$$

Under a fixed quantity restriction, the counterparts of the above two equations are:

$$J_Q(q_0) = \max_{X} \int_0^\infty e^{-rt}[B(X(p)) - D(q_t)]dt + e^{-rt}V(q_T), \qquad (3.17)$$

$$\text{subject to } \dot{q}_t = \alpha X(P) - \delta q_t,$$

$$\int_0^\infty e^{-rt}\left[\frac{dB(X)}{dX} - \frac{dD(q_t)}{dq_t}\frac{\partial q_t}{\partial X}\right] = 0. \qquad (3.18)$$

The common factor dX/dp of the integrand in Equation (3.16) represents the change of the polluter's marginal response to the tax rate. For a given tax rate p, the polluter will determine the emission level according to the condition that $B_1(X) = p$. Using the implicit function rule, it follows that $dX/dp = 1/B_{11}(X)$. Since the tax rate p stays at a certain fixed level under a one-time control tax policy, the level of emission X is also to be fixed at a constant level. Consequently, the value of function $B_{11}(X)$ is fixed at a constant level and the common factor dX/dp can be dropped out of the equation. Therefore, Equation (3.16) is equivalent to Equation (3.18), and both equations provide the same information and yield the same solution of X provided that the initial conditions and constraints are the same. The equivalence can be denoted in the following form:

$$X(\tilde{p}) = \hat{X} \tag{3.19}$$

where $X(p)$ is the emission rate for a given tax rate p, $\tilde{p}$ is the solution of Equation (3.16), and $\hat{X}$ is the solution of Equation (3.18).

As has been indicated in relation with Equation (3.14), under both of the control modes with fixed control variables, q_t will approach asymptotically to the steady state point $\alpha\hat{X}/\delta$. However, it is not yet clear whether the steady state points of these fixed control modes will be greater than, equal to, or less than that of the first-best solution. In order to show the relative positions of these different steady state points, we need to take several steps. Since the equivalence of the two second-best policies is already shown, the comparisons will be made only with the first-best solution and the fixed quantity restriction in order to avoid repetition of the same statement.

The following proposition is apparent:

Proposition 2

$$V(q_0)^{10} \geq J_Q(q_0).$$

The above inequality relationship between the two maximized functions is a definitional problem and simply means that the first-best policy cannot be inferior to the second-best one.

Suppose that the initial state q_0 is equal to q_s^*. Then, the convergence assumption in the previous section tells us that the solution of $\{X_t^*\}$ of equation (3.8) is X_s^*. Comparison of the right hand side terms of equations (3.8) and (3.16) indicates that these two functions $V(\cdot)$ and $J_Q(\cdot)$ yield the same value if both functions take the same values of q_0 and X_t for all the planning horizon. Since X_s^* is a constant, the control variable of the second best policy can be chosen in such a way that $\hat{X} = X_s^*$. This implies that $J_Q(q_0)$ can be at least as great as $V(q_0)$, *i.e.*, $V(q_s^*) \leq J_Q(q_s^*)$. But in Proposition 2, the possibility of the inequality relationship $V(q_0) < J_Q(q_0)$ has been excluded. Consequently, it follows that $V(q_s^*) = J_Q(q_s^*)$, which is true if and only if $\hat{X} = X_s^*$. This result can be maintained as follows:

Proposition 3

$$\text{If } q_0 = q_s^*, \text{ then } V(q_0) = J_Q(q_0) \text{ and } \hat{X} = X_s^*$$

The meaning of this proposition is straightforward and does not seem to require much explanation. If the initial state of the stock coincides with the steady state

[10]The definition of $V(q_0)$ is given in equation (3.8).

of the first-best policy, then there is no difference between the first-best and the second-best policies because both policies will have the same solution.

Taking a total derivative of Equation (3.18) with respect to $\hat{X}$ and q_0 and rearranging the terms, it can be shown that $d\hat{X}/dq_0 < 0$,[11] whose derivation is shown in Section A of Appendix. The positive linear relationship between $\hat{q}_s$ and $\hat{X}(\hat{q}_s = \alpha\hat{X}/\delta)$ indicates that $\partial\hat{q}_s/\partial q_0 < 0$, which leads directly to the following proposition.

Proposition 4 *A higher level of initial state q_0 will be associated with a lower level of steady state $\hat{q}_s$ of the fixed one-time control policy.*

As an immediate result of Proposition 3 and Proposition 4, the following corollary can be stated.

Corollary 1 *If the initial level q_0 is higher (lower) than the first-best steady state q_s^*, then the steady state level of the second-best control $\hat{q}_s$ is lower (higher) than q_s^*.*

Figure 3.2 shows how the difference in the initial states q_0's can affect the states in the subsequent periods. According to Proposition 2, the horizontal line $q_t = q_s^*$ represents the controlled level of q_t when $q_0 = q_s^*$, under the first-best optimal control as well as second-best controls. If the initial level q_0 is higher (lower) than q_s^*, such as q_0' (q_0''), q_t will approach $\hat{q}_s'$ ($\hat{q}_s''$) from above (below), where $\hat{q}_s' < q_s^*$ ($\hat{q}_s'' > q_s^*$).

Even though this result might appear somewhat unusual, a short intuitive explanation can clarify the reason. With the presence of a positive discounting rate, the controller's decision making will reflect the higher weight of its effect on

[11]The intuitive explanation of this result is given in the later part of this section.

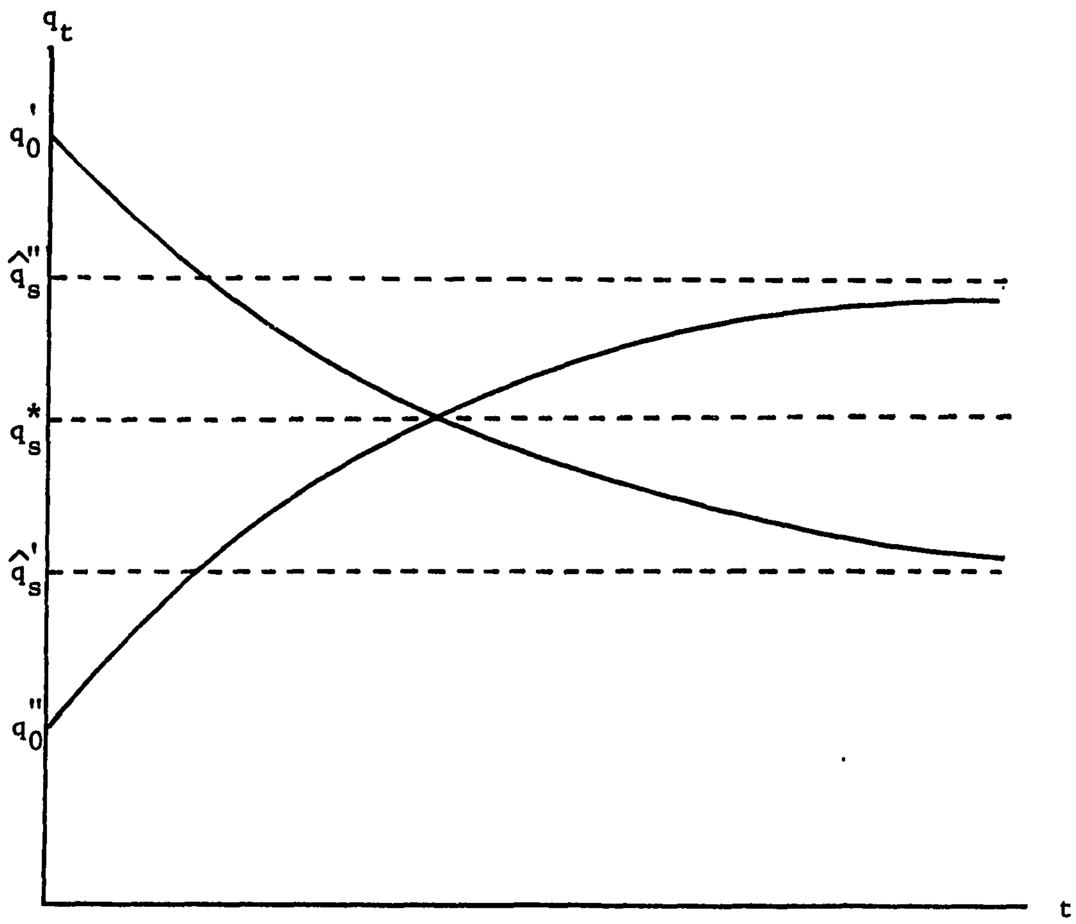

Figure 3.2: Dynamic path of one-time control

the immediate near future as compared to that of the distant future. If the initial state of the stock is not very deteriorated, then a higher rate of emission does not change the state of the stock so adversely as to raise any serious damage at least for a sufficient length of period. Such a decision will lead to a highly contaminated steady state of $\hat{q}_s$ in the long run. On the other hand, if the stock is much contaminated at the initial stage, then a more drastic reduction of the emission is desired for the improvement of the state immediately and the steady state resulting from this reaction will be a very low level of contamination.

The preceding explanations are only about the long-term behaviors of the fixed one-time control policy and does not explain the relative positions of the first-best and second-best policies in the near future. A complete comparison of the paths associated with the first-best and the second-best policies will be possible when the levels of emission of these two policies are compared.

It is possible to prove that the controlled emission level under a second-best policy ($\hat{X}$) falls into the interval whose bounds are determined with the first-best optimal emission level at the beginning of the control (X_0^*) and the first-best steady state emission level (X_s^*). For the purpose of the proof, let us define a function $j(\cdot)$, such that, for a given q_0,

$$j(q_0) = \int_0^\infty e^{-rt}[B(X) - D(q_t)]dt, \tag{3.20}$$

$$\text{subject to } \dot{q}_t = \alpha X - \delta q_t.$$

Function $j(q_0)$ is not a maximized one. If and only if X is chosen to be $\hat{X}$, then the value of $j(q_0)$ is equivalent to those of $J_Q(q_0)$ and/or $J_p(q_0)$. In other words, if

X is chosen to be $\hat{X}$, then the value of $dj(q_0)/dX$ will be identical to zero. On the other hand, due to the concavity of the function $j(\cdot)$, if $dj(q_0)/dX$ is evaluated at some X_c, where $X_c > \hat{X}$, then $dj(q_0)/dX$ takes a negative value; if $X_c < \hat{X}$, then $dj(q_0)/dX$ takes a positive value. In what follows, the signs of $dj(q_0)/dX$ will be determined at the potential critical points X_0^* and X_s^*.

Since the control variable X is fixed throughout all the future period, $B_1(X)$ is also constant and $dj(q_0)/dX$ can be expressed in a simpler form.

$$\frac{dj(q_0)}{dX} = \frac{B_1(X)}{r} - \frac{\alpha}{\delta}\int_0^\infty D_1(q_t)e^{-rt}(1 - e^{-\delta t})dt, \tag{3.21}$$

$$\text{where } \dot{q}_t = \alpha X - \delta q_t$$

When $dj(q_0)/dX$ is evaluated at X_0^*,

$$\frac{dj(q_0)}{dX}|_{X=X_0^*} = \frac{B_1(X_0^*)}{r} - \frac{\alpha}{\delta}\int_0^\infty D_1(q_t^0)e^{-rt}(1 - e^{-\delta t})dt, \tag{3.22}$$

$$\text{where } \dot{q}_t^0 = \alpha X_0^* - \delta q_t$$

In footnote 9 of Section 3.2, λ_t is given in an explicit form. Combining this solution of λ_t and Equation (3.4), and setting $t = 0$, we can show that:

$$B_1(X_0^*) = \alpha \int_0^\infty e^{-(r+\delta)t} D_1(q_t^*)dt. \tag{3.23}$$

Substituting the above equation into (3.22),

$$\frac{dj(q_0)}{dX}|_{X=X_0^*} = \frac{\alpha}{r}\int_0^\infty [D_1(q_t^*)e^{-(r+\delta)t} - \frac{r}{\delta}D_1(q_t^0)(e^{-rt} - e^{-(r+\delta)t})]dt. \tag{3.24}$$

Let us first consider the case where the initial state q_0 is less deteriorated than the optimal steady state q_s^*, which is $q_0 < q_s^*$. Proposition 1 in the previous Section tells us that $X_0^* \geq X_t^*$ and $q_t^0 \geq q_t^*$ for all t, where the equality signs hold only when $t = 0$ for both cases. From this, it follows that $D_1(q_t^0) > D_1(q_t^*)$ when $t > 0$ and $D_1(q_t^0) = D_1(q_t^*)$ for $t = 0$. Therefore, Equation (3.24) can be changed into an inequality relationship such that

$$\frac{dj(q_0)}{dX}|_{X=X_0^*} < \frac{\alpha}{r\delta}\int_0^\infty D_1(q_t^*)[(r+\delta)e^{-(r+\delta)t} - re^{-rt}]dt. \tag{3.25}$$

Integrating by parts,

$$\frac{dj(q_0)}{dX}|_{X=X_0^*} < \frac{\alpha}{r\delta}\left[D_1(q_t^*)(e^{-rt} - e^{-(r+\delta)t})|_0^\infty - \int_0^\infty D_{11}\dot{q}_t^*(e^{-rt} - e^{-(r+\delta)t})dt\right] < 0. \tag{3.26}$$

Since q_t^* has an upper bound, $D_1(q_t^*)$ is also finite, so the first term inside of the brace drops out when it is evaluated both at $t = 0$ and $t = \infty$. The sign of (3.26) is determined by the second part where the sign of the integrand is positive because $D_{11} > 0$, and $\dot{q}_t^* > 0$.[12] Consequently, $dj(q_0)/dX$ takes a negative value when evaluated at $X = X_0^*$. This proves that the fixed level of emission under a second-best policy, $\hat{X}$, is less than X_0^* when q_0 is less than q_s^*.

Furthermore, when $dj(q_0)/dX$ is evaluated at X_s^*,

$$\frac{dj(q_0)}{dX}|_{X=X_s^*} = \frac{B_1(X_s^*)}{r} - \frac{\alpha}{\delta}\int_0^\infty D_1(q_t^1)e^{-rt}(1 - e^{-\delta t})dt, \tag{3.27}$$

where $q_t^1 = \alpha X_s^* - \delta q_t$.

[12] If $q_0 < q_s^*$, then $\dot{q}_t^* > 0$, which has been mentioned in Proposition 1.

By assumption, $q_0 < q_s^*$, so $q_t^1 < q_s^*$ for all $t > 0$. Hence, when $\hat{X} = X_s^*$, Equation (3.23) is representable in such a simpler form as

$$B_1(X_s^*) = \frac{\alpha D_1(q_s^*)}{r+\delta}. \tag{3.28}$$

Substituting Equation (3.28) into (3.27) and utilizing the inequality relationship between q_t^1 and q_s^*, the following inequality relationship can be constructed.

$$\frac{dj(q_0)}{dX}|_{X=X_s^*} > \frac{\alpha D_1(q_s^*)}{r(r+\delta)} - \frac{\alpha D_1(q_s^*)}{\delta}(\frac{1}{r} - \frac{1}{r+\delta}) = 0. \tag{3.29}$$

The above inequality implies that $\hat{X} > X_s^*$. This completes the proof that $X_s^* < \hat{X} < X_0^*$ when $q_0 < q_s^*$.

Simply following the same procedure, it can be shown that $X_0^* < \hat{X} < X_s^*$ when $q_0 > q_s^*$. Since Equation (3.24) is true for any value of q_0, the inequality relationship that $X_0^* \leq X_t^*$ in the case of $q_0 > q_s^*$ can be substituted into Equation (3.24) to establish the inequality condition that

$$\frac{dj(q_0)}{dX}|_{X=X_0^*} > \frac{\alpha}{r\delta}\int_0^\infty D_1(q_t^*)[(r+\delta)e^{-(r+\delta)t} - re^{-rt}]dt. \tag{3.30}$$

Since $\dot{q}_t^* < 0$ when $q_0 > q_s^*$ as given in Proposition 1, the sign is determined such that

$$\frac{dj(q_0)}{dX}|_{X=X_0^*} > \frac{\alpha}{r\delta}\left[D_1(q_t^*)(e^{-rt} - e^{-(r+\delta)t})|_0^\infty - \int_0^\infty D_{11}\dot{q}_t^*(e^{-rt} - e^{-(r+\delta)t})dt\right] > 0. \tag{3.31}$$

This proves that $\hat{X} > X_0^*$ when $q_0 > q_s^*$.

Since X_s^* is the upper bound of the series of $\{X_t^*\}$ when $q_0 > q_s^*$, we know that $q_t^1 > q_s^*$ for all $t > 0$. Hence, it can be shown that

$$\frac{dj(q_0)}{dX}|_{X=X_s^*} < \frac{\alpha D_1(q_s^*)}{r(r+\delta)} - \frac{\alpha D_1(q_s^*)}{\delta}(\frac{1}{r} - \frac{1}{r+\delta}) = 0. \tag{3.32}$$

The two inequalities (3.31) and (3.32) prove that $X_0^* < \hat{X} < X_s^*$ when $q_0 > q_s^*$.

Combining Proposition 2 with these findings, the following statement is possible.

Proposition 5 *If the initial level q_0 differs from q_s^*, then the optimal emission level $\hat{X}$ under one-time control policy falls into the open interval whose bounds are determined by X_0^* and X_s^*, where X_0^* and X_s^* represent the first-best optimal emission level and first-best steady state emission level, respectively; if $q_0 = q_s^*$, then $\hat{X} = X_s^*$.*

Even though the proof is somewhat complicated, the underlying intuition of the preceding proposition is straightforward. In a fixed one-time control, the controller has to pick one number that can best approximate the first-best path $\{X_t^*\}$ with a proper weight over the planning horizon. This means that the fixed control level should be a weighted average of the first-best path of X_t^* for the corresponding period from 0 to infinity, where the weight is determined by all of the components of the objective function, such as the level of discount, the forms of both benefit and damage functions, and the equation of motions, etc. In other words, even though it is difficult to determine how the weights are distributed, the fact is that $\hat{X}$ is a certain form of a weighted average of X_t^*. It implies that $\hat{X}$ cannot be outside of the boundaries determined by X_s^* and X_0^*.

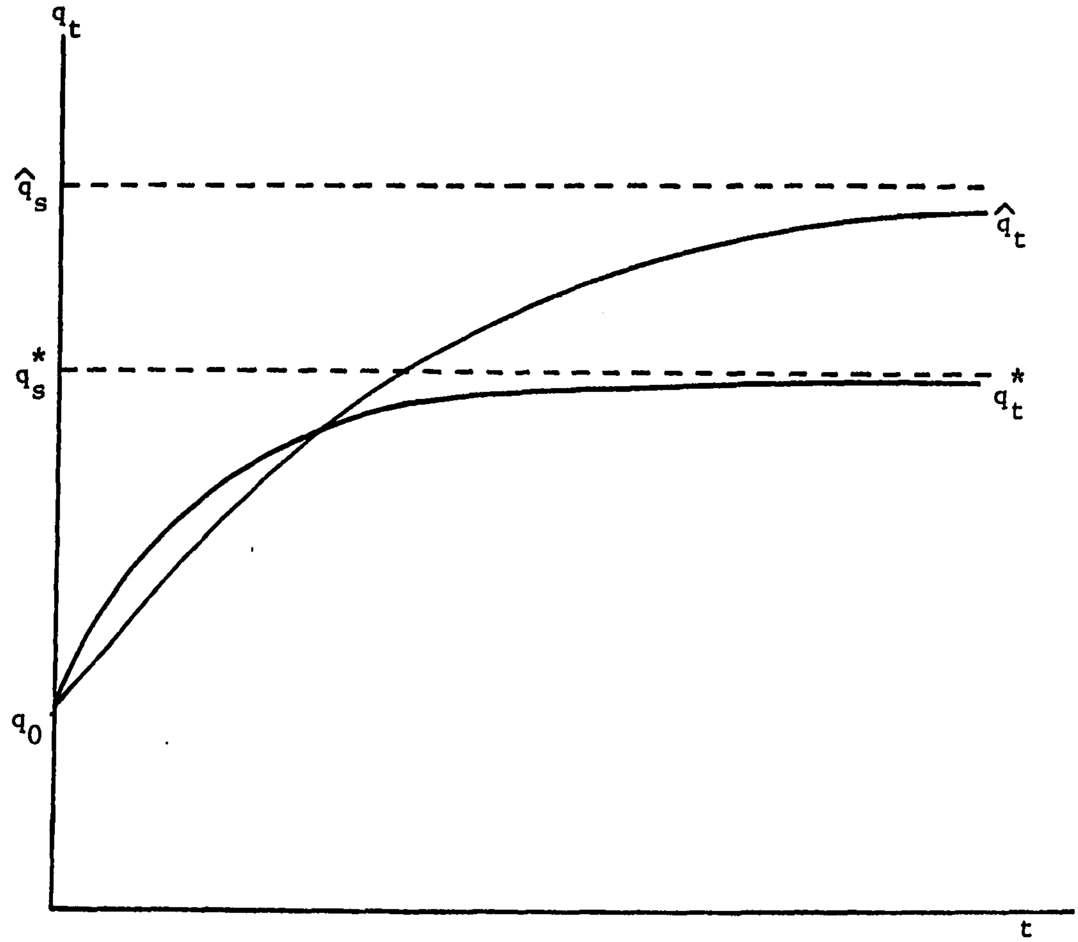

Figure 3.3: Comparison of the dynamic paths: optimal control and one-time control

The above comparison on the emission levels under the first-best and the second-best one-time control policies makes it possible to show the complete time paths of the stock variables associated with different policies. Starting from the same initial level of q_0, where $q_0 < q_s^*$, q_t^* will be higher than $\hat{q}_t$ at the early stage because X_t^* is greater than $\hat{X}$ in the first place. As X_t^* becomes lower than $\hat{X}$, $\hat{q}_t$ catches up with q_t^* and eventually $\hat{q}_t$ will surpass q_t^*. Since the level of stock is the accumulation of the emission, the time when q_t^* intersects $\hat{q}_t$ lags behind the time when X_t^* intersects $\hat{X}$. Figure 3.3 shows the change in the relative positions of the state variables under the first-best policy and the second-best case.

4 OPTIMAL COMMITMENT TIME IN ONE-TIME CONTROL

4.1 Introduction

One of the casual observations on the externality problem is that it does not receive sufficient attention until it begins to raise very serious problems. This may be taken for granted reflecting the fact that an externality implies independent decision making by an economic agent without any proper consideration of the effect of his economic behavior on a third party. However, in some cases, long-time negligence of the problem until it becomes a critical issue would be accompanied by a high social external cost, which would otherwise have been reduced with an earlier intervention of the government.

In the case of a static flow externality problem, a late commitment may not raise such a great efficiency loss. If a proper measure is in place, even in a later time, the environmental condition can be recovered to the desirable state instantaneously and any additional loss in the subsequent period can be avoided. However, in a stock externality problem, even drastic policy measures cannot change the state immediately. Consequently, a great economic loss is inevitable if the control policy is not implemented at a proper time.

Probably, a delayed intervention of the controlling authority in an externality problem may not necessarily be attributed only to the inertia of the bureaucracy.

There are a number of restrictions in implementing proper measures on an externality problem, especially in good times. In some cases, the significance of the problem may not have been anticipated before it has fully matured. In other cases, an institutional limitation or social customs may block the introduction of a proper control policy.

Considering some of these non-economic factors as given conditions, a fundamental question can be raised: is it true that, on purely economic grounds, an earlier commitment in the control of stock externality is always more desirable than a later engagement? If that is not always the case, then when is the optimal starting point of control? These are the main issues that will be dealt with in this chapter, which is a direct extension of the preceding chapter in the fact that all the basic assumptions of the last chapter will be retained.

As an intuitive explanation on the questions raised above, the following fact can be indicated. When there is a difference in the initial states, and all the rest of the conditions are identical, then the paths of one-time fixed controls corresponding to these initial conditions look like those shown in Figure 4.1. It is true that $J_Q(q_0^1) > J_Q(q_0^2)$ as far as $q_0^1 < q_0^2$, since a higher level of contamination of the initial state of the stock will always have a greater adverse effect on the net social benefit, *ceteris paribus*. However, the path taken by the second-best policy is closer to that of the first-best one as the initial level q_0 gets closer to qs^*. And in the extreme case of $q_0 = q_s^*$, the path of the second-best policy is the same as that of the first-best policy. This indicates that the difference in efficiency between the first-best policy and the second-best policy gets smaller as q_0 comes closer to q_s^*.[1] This

[1]It is true that the divergence from the path of the first-best solution cannot

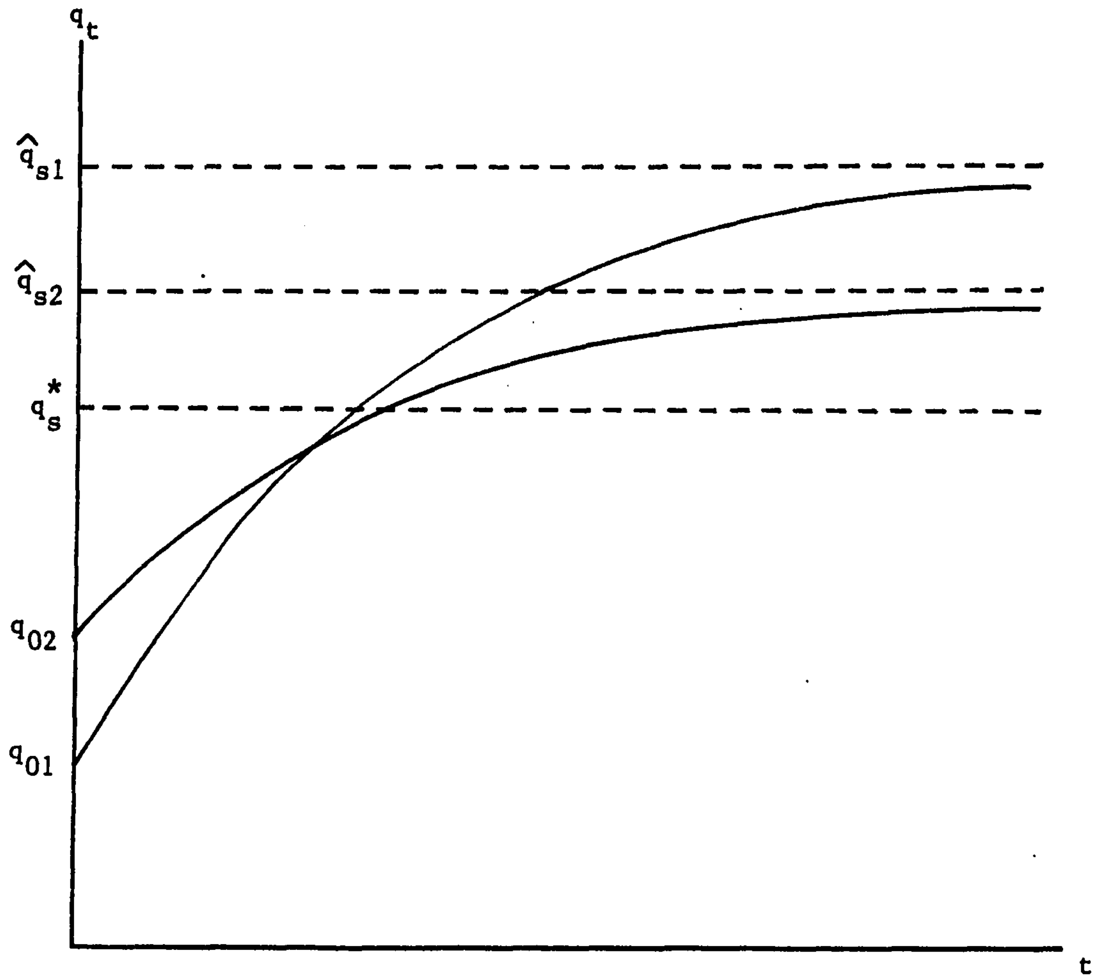

Figure 4.1: **The relationships between initial states and corresponding second-best dynamic paths**

shows the possibility that there can exist certain conditions under which deferring the implementation of a fixed control on an emission yields some efficiency gain. However, such a conjecture is never relevant in the case of first-best policy. If the control variable is freely changeable, then the option of no-control can be a subset of the controller's decision making. Hence, an earlier commitment in the control cannot be inferior to a later commitment under any circumstances.

In order to describe it more specifically, first of all, let us assume that the controller can choose the starting point of time[2] in the control of an externality. In some sense, this condition means to provide an additional option to the controlling authority. Figure 4.2 shows how this additional option can allow the controller to approximate the path of the first-best policy under the restriction of a perfect rigidity of the policy measures.

In Figure 4.2, all the four q's with subscript s along the vertical axis represent the steady state levels associated with different control decisions including the no-control state. Suppose that the policy can be implemented from time 0 and the state at this moment is q_0. In the figure, the curve q_0a that approaches $\bar{q}_s$ represents the level of $\bar{q}_t$ under no control, where $\bar{q}_s = \alpha\bar{X}/\delta$ and $\bar{X}$ satisfies the condition that $B_1(\bar{X}) = 0$. Curve q_0c represents the change of q_t under the first-best policy. The curve q_0b, on the other hand, is the path associated with the fixed control policy that is to be implemented from $t = 0$.

be an exact measure of the economic loss associated with a non-first-best solution because the benefit or the damage functions are non-linear and under a positive discount factor, the benefit or damage of each period does not receive an equal weight.

[2]This idea has some analogy with a free-time optimal control problem. For the details of a free-time optimal control, refer to Kamien and Schwartz (1981, pp. 143-149).

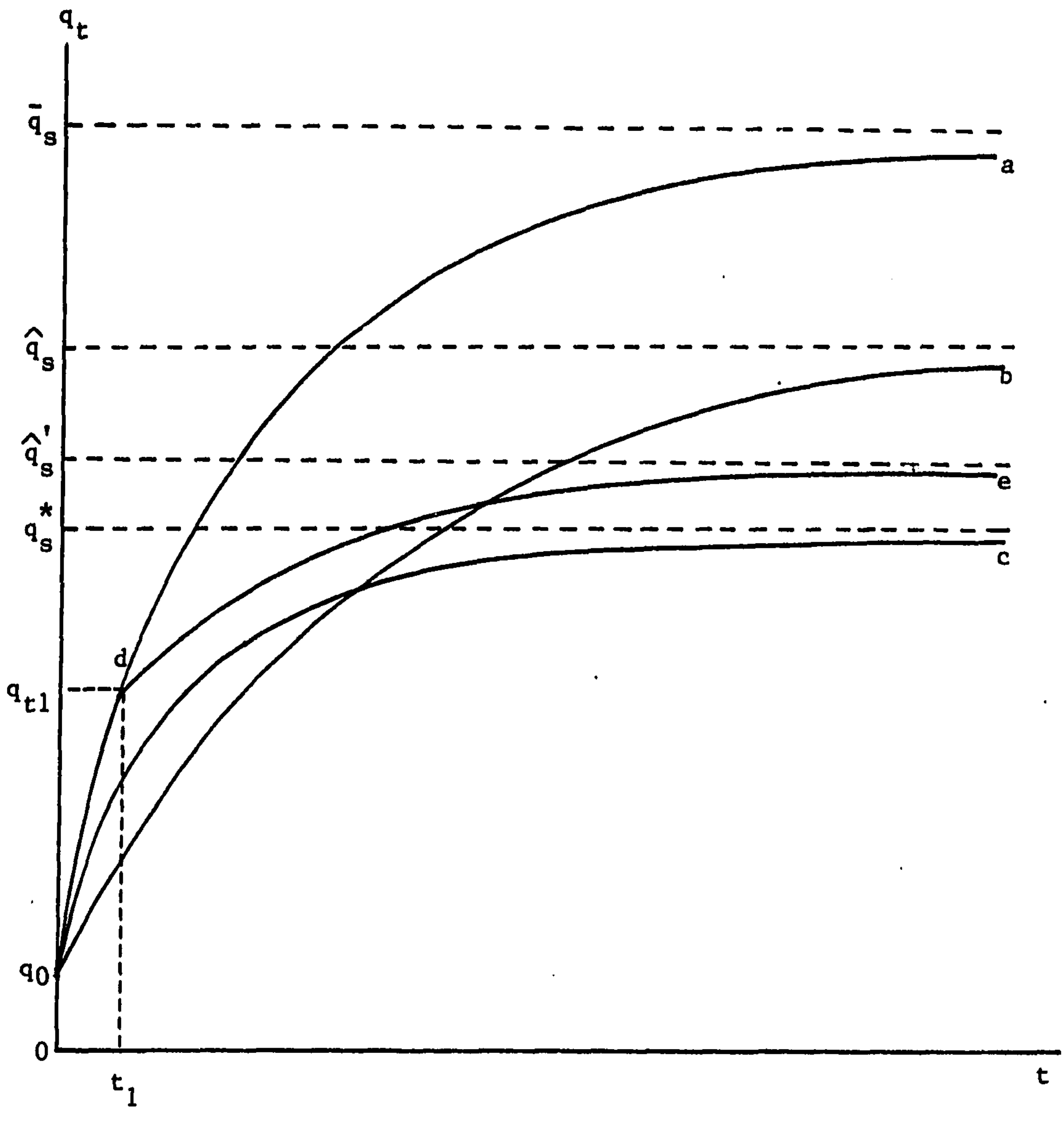

Figure 4.2: Deferral in a control and the change in dynamic path

The relative positions of these three curves q_0a, q_0b and q_0c are drawn such that q_0a lies above all the rest of the curves throughout all the period subsequent to the initial time 0, because this one is associated with a no-control policy. As for the paths of q_0b and q_0c, it has already been mentioned in the explanation of Figure 3.3, at the end of the previous chapter.

As shown in Figure 4.2, if the starting time of control is deferred from 0 to t_1, then the change of q_t, in this particular case, will take the path represented with the kinked curve q_0de. Until the time of t_1, there will be no control and the level of q_t is on that part of the q_0a curve which is corresponding to no-control policy. Since the initial level q_t^1 is higher than q_0, according to Corollary 1 in the previous chapter, the delayed control will be associated with a lower level of steady state than the early commitment case. In Figure 4.2, the curve q_0de is in a closer distance to the first-best optimal path q_0c than the curve q_0b for most of the period of time. This observation allows us to contemplate the possibility that a delayed start of a control like the case shown with the curve q_0de might have greater efficiency than the case of earlier commitment in control as is represented with the curve q_0b. In the subsequent section, a decision rule for choosing the optimal starting point is to be derived.

4.2 Decision Rule for the Optimal Commitment Time

In order to incorporate the starting point of control as an additional argument, let us redefine $J(\cdot)$ temporarily[3] in the following manner:

[3]Eventually, we drop this definition and resume the simple original definition.

$$J(q_m, t_m) = \max \int_{t_m}^{\infty} e^{-rt} \left[B(X) - D(q(q_m, t - t_m)\right] dt, \tag{4.1}$$

$$\text{subject to } \dot{q}_t = \alpha X - \delta q_t, \quad \text{where } t \geq t_m.$$

In the maximized function $J(q_m, t_m)$, the first argument q_m represents the state of the stock at the moment of starting the control, while the second argument, t_m, indicates the chronological date at which the control starts. Therefore, both $J_p(q_0)$ or $J_Q(q_0)$ defined in the preceding chapter are a special form of $J(q_m, t_m)$ with the condition that q_m and t_m are set to be q_0 and 0, respectively. The controlled state of the stock can be defined with two arguments: the state of the stock at the starting point of control (q_m) and the length of control up to that moment ($t - t_m$).[4]

Realizing that $J(q_m, t_m)$ is an autonomous system, (4.1) can be re-expressed in a simpler form:

$$\begin{aligned} J(q_m, t_m) &= \max \int_{t_m}^{\infty} e^{-rt} \left[B(X) - D(q(q_m, t - t_m))\right] dt \\ &= e^{-rt} J(q_m, 0) \equiv e^{-rt} J(q_m). \end{aligned} \tag{4.2}$$

The above representation implies that if the initial states of the stocks are the same, then there is no difference in the controlled level of the emission. For example, the dynamic path associated with the maximized function $J(q_m, t_m)$ is exactly the same as the path taken by $J(q_m, 0)$ except the time lag of t_m. That is why $J(q_m, t_m)$ can be expressed with $J(q_m, 0)$ only by discounting for the lagged time period t_m. Since

[4] A full description of the state of a stock requires the fixed emission level X as an argument, but X is the choice variable and its the optimal value $\hat{X}$ is also an endogenous variable that is determined by q_m, so X can be suppressed.

the second argument of $J(\cdot)$ can be set to be zero, the original form of maximized function $J(q_m)$ can be substituted for the cumbersome notation $J(q_m, 0)$.

Let us set the chronological date 0 to be the moment at which the decision on the control is to be made and t_m to be the date at which the control actually starts, where $t_m \geq 0$. The decisions that have to be made by the controlling authority are when to start the control and how much emission to allow. With a known initial condition of the stock at the decision moment (q_0), the state of the stock at the control starting moment (q_{t_m}) is fully described with the length of delay in control (t_m). For a given state of the stock at the beginning of control (q_{t_m}), the optimal level of fixed control $\hat{X}$ is also determined. Therefore, choosing the optimal starting point of time for the control is the only decision that the controller has to make and all the other variables like $\hat{X}$ and $\hat{q}_t$ are automatically determined.

If the controller is given the option to choose the starting point of time, his objective function is to maximize the discount benefit of those periods under no-control which last from $t = 0$ to $t = t_m$, as well as the discounted benefit under the controlled state which continues after $t = t_m$. In order to formulate the question more clearly, let us introduce a new functional form $\mathcal{J}(\cdot)$, which is defined in such a way that:

$$\mathcal{J}(q_0) = \max \int_0^{t_m} e^{-rt} \left[B(\bar{X}) - D(q_t)\right] dt + e^{-rt_m} J(q_{t_m}, (q_0, t_m)), \qquad (4.3)$$

$$\text{subject to } \dot{q}_t = \alpha X - \delta q_t, \quad \text{where } X = \begin{cases} \bar{X} & \text{for } 0 \leq t \leq t_m, \\ \hat{X} & \text{for } t \geq t_m, \end{cases}$$

$$\text{and } q_0 \text{ is given.}$$

The first term on the right hand side of (4.3) represents the discounted benefit that will continue from $t = 0$ through $t = t_m$, and the second term corresponds to the maximal value of the discounted benefit under the controlled state that will continue throughout all the period subsequent to $t = t_m$. In (4.3), the argument of the maximized function $J(\cdot)$ is q_{t_m}, which is in turn, expressed with two arguments, q_0 and t_m. What is meant by these two arguments of q_{t_m} is that the state of the stock at the starting point of control (q_{t_m}) is completely described with the initial state of the stock at $t = 0$, which is q_0, and the length of time during which the control policy has not been implemented, which is t_m. In other words, by substituting $t = t_m$, $X = \bar{X}$, and a specific value of q_0 into Equation (3.14), q_{t_m} is uniquely determined.

In finding the optimal starting point of control, the first-order condition is

$$\frac{\partial \mathcal{J}(q_0)}{\partial t_m} = e^{-rt_m}\left(\left[B(\bar{X}) - D(q_{t_m})\right] + \frac{\partial J(q_{t_m})}{\partial q_{t_m}}\frac{\partial q_{t_m}}{\partial t_m}\right) \leq 0, \tag{4.4}$$

where $q_{t_m} \geq q_0$ or $t_m \geq 0$. As represented by the notation $\partial \mathcal{J}/\partial t_m$ on the left hand side, the terms on the right hand side of Equation (4.4) represent the trade-off associated with the deferral of the control for a unit period of time. Since all the terms inside the brace are expressed in the current value at the potential starting moment of control which is some unknown time t_m in the future, all the values should be discounted by e^{-rt_m} in order to express in terms of the values at the decision moment, which is denoted as $t = 0$. The first term in the brace, $[B(\bar{X}) - D(q_{t_m})]$, represents the possible static net benefit when the control is deferred for a unit period of time. Any dynamic decision will affect the state of those periods subsequent to the decision moment. In this particular case, such dynamic effects can be captured by the last two terms in the brace: the term $-rJ(q_{t_m})$ is the pure delayed cost; the other term $[(\partial J/\partial q_{t_m})(\partial q_{t_m}/\partial t_m)]$ represents the cost associated with a higher level of q_{t_m} than would have been the case with the earlier commitment. Since the term $\partial q_{t_m}/\partial t_m$ represents the rate of change in q_{t_m} during the time of deliberate delay of a control on the emission, which is in a no-control state, the change of q_t follows the rule of motion that $\dot{q}_t = \alpha\bar{X} - \delta q_t$. If q_{t_m} is known, then the rate of change of q_{t_m} is

$$\frac{\partial q_{t_m}}{\partial t_m} = -\delta(q_{t_m} - \frac{\alpha\bar{X}}{\delta}) = -\delta(q_0 - \frac{\alpha\bar{X}}{\delta})e^{-\delta t_m}. \tag{4.5}$$

Finally, by substituting equation (4.5) into (4.4), the first-order condition becomes

$$\frac{\partial \mathcal{J}(q_0)}{\partial t_m} = e^{-rt_m}\left(\left[B(\bar{X}) - D(q_{t_m})\right] + \delta(\bar{q}_s - q_{t_m})J_1(q_{t_m}) - rJ(q_{t_m})\right) \leq 0, \tag{4.6}$$

$$\text{where } t_m \geq 0.$$

The second-order condition is

$$\frac{\partial^2 \mathcal{J}(q_0)}{\partial t_m^2} < 0. \tag{4.7}$$

An example will show more specifically how the above result would work in a particular case. For simplicity of calculation, a quadratic benefit function and a linear damage function[5] are assumed:

$$B(X_t) \equiv b_1 X_t - \frac{b_2}{2}X_t^2, \tag{4.8}$$

[5]In Chapter 3, a strictly convex damage function is assumed. But a linear damage function still fulfills the condition for the concavity of function $J(q_0)$, and simplifies the calculation to a greater extent.

$$D(q_t) \equiv d_1 q_t,$$

where b_1, b_2 and d_1 are all positive, and $b_1(r+\delta) > d_1$. For a further simplification, we can set the parameter α to be 1 without losing any generality.[6] If the control starts at $t = 0$, with a given initial state q_0, then the optimal level of emission is

$$\hat{X} = \frac{b_1(r+\delta) - d_1}{b_2(r+\delta)}. \tag{4.9}$$

By substituting this optimal value $\hat{X}$ into the objective function, which is Equation (3.17) in the previous chapter, the maximized function $J(q_0)$ is derived.

$$J(q_0) = \frac{[b_1(r+\delta) - d_1]^2}{2b_2 r(r+\delta)^2} - \frac{d_1 q_0}{r+\delta}. \tag{4.10}$$

All the required information for the other terms in Equation (4.6) can be derived from the given assumptions, *i.e.*,

$$\bar{X} = \frac{b_1}{b_2}, \tag{4.11}$$

$$B(\bar{X}) = \frac{B_1^2}{2b_2},$$

$$D(q_t) = d_1 \left[\left(q_0 - \frac{b_1}{b_2} \right) e^{-\delta t_m} + \frac{b_1}{b_2} \right].$$

Substituting all of this information into Equation (4.6), considering that q_{t_m} is also a function of t_m and q_0, the optimal starting point of time is determined, such that

[6]This can be done by reparameterizing q_t.

$$\text{when } 0 \leq q_0 < \frac{4b_1(r+\delta) - d_1}{4b_2(r+\delta)}, \; t_m = \frac{\log 4b_2(r+\delta) + \log(b_1/b_2 - q_0) - \log d_1}{\delta}, \tag{4.12}$$

$$\text{and when } \frac{4b_1(r+\delta) - d_1}{4\delta b_2(r+\delta)} \leq q_0 < \bar{q}_s, \; t_m = 0.$$

The advantage associated with postponement of a control is shown more evidently when specific values are plugged in. For example, if $b_1 = 1, b_2 = 1, d_1 = 0.05, r = 0.05, \delta = 0.2, \alpha = 1$, and $q_0 = 2.5$, then the solution for the optimal starting point is $t_m = 11.5$. This means that the fixed control should be deferred for 11.5 unit periods of time when the given initial state q_0 is at 2.5. Suppose that the controller has erroneously decided to start a fixed control policy at the initial moment. Then the discounted net future benefit $J(2.5) = 5.9$. On the other hand, if the control policy has been deferred for the proper period of time, then the discounted net future benefit $\mathcal{J}(2.5) = 8.1$. By delaying the control for 11.5 unit periods of time, the net discounted benefit has been improved by the difference between the two maximized values $\mathcal{J}(2.5)$ and $J(2.5)$, which is 2.2.

As a summary of this section, the following proposition is presented.

Proposition 6 *If a policy variable cannot be changed freely, then an earlier start of a stock externality control may not necessarily be desirable.*

Since the proposition is in a partial negation, the example given above is sufficient for the proof.

4.3 Maximum Delay and Administrative Cost

Even though the result summarized in Proposition 6 is an interesting one, it will not be easy to determine an optimal starting point in a real world problem. The complexity of the decision rule given in (4.6) actually limits its applicability to a practical purpose. Therefore, if we can specify a bound in the initial state q_0 beyond which the benefit from delaying a control is not possible, then it will be of great help in actual decision making. Such a finding can save the doubts that may otherwise have been cast upon the starting points of control.

The basic intuition that made us expect the existence of the optimal starting point is shown in the graphical representation in Figure 4.2. What we observe in the Figure is that, when the initial state q_0 is lower than that of the first-best steady state, a deferral of control may lead the overall dynamic path closer to that of the first-best policy. On the contrary, as far as the graphical representation is concerned, when the initial state q_0 is higher than that of the first-best steady state, any deferral of the control seems to make its dynamic path more divergent from the path associated with the first-best policy. Therefore, a possible question is whether there can be any economic benefit of deferral even when the initial state is more deteriorated than the first-best steady state.

As for this question, one important piece of information is available from Proposition 3, where it was shown that the second-best policy is equivalent to the first-best one and that the optimal level of emission $\hat{X}$ should be equal to X_s^* if the initial state of the stock happens to coincide with the level of the first-best steady state, *i.e.*, $q_0 = q_s^*$. Since no other policy can improve upon the first-best policy, it automatically follows that no extra net benefit can be expected from any delay of control, which implies that the first-order condition in (4.6) is fulfilled at $t_m = 0$. In a mathematical notation, this can be restated such that

$$\frac{\partial \mathcal{J}(q_s^*)}{\partial t_m}|_{t_m=0} = B(\bar{X}) - D(q_s^*) - \delta(\bar{q}_s - q_s^*)J_1(q_s^*) - rJ(q_s^*) \leq 0. \tag{4.13}$$

As has been illustrated in Figure 3.2, when $q_0 > q_s^*$, the change of the state q_t is decreasing, *i.e.*, $\dot{q}_t < 0$. This allows us to construct the following inequality relationship.[7]

$$\frac{D_1(q_0)}{r+\delta} > -J_1(q_0), \text{when } q_0 > q_s^*. \tag{4.14}$$

Based on this information, we can derive the following result:

$$\frac{\partial}{\partial q_0}\left[\frac{\partial \mathcal{J}(q_0)}{\partial t_m}|_{t_m=0}\right] = -D_1(q_0) - (r+\delta)J_1(q_0) + \delta(\bar{q}_s - q_0)J_{11}(q_0) < 0, \tag{4.15}$$

$$\text{if } q_0 > q_s^*.$$

The sign of the above equation has been determined due to the fact that $-D_1(q_0) - (r+\delta)J_1(q_0) < 0$, when $q_0 > q_s^*$ and that $J_{11}(q_0) < 0$.[8] What (4.15) implies is that if the initial state q_0 is greater than q_s^*, then a greater value of q_0 will be associated with a lower value of $\partial \mathcal{J}(q_0)/\partial t_m$ at $t_m = 0$. Therefore, from the two inequalities (4.13) and (4.15), it follows that

$$\frac{\partial \mathcal{J}(q_0)}{\partial t_m}|_{t_m=0} < 0, \text{ for all } q_0, \text{ where } q_0 > q_s^*. \tag{4.16}$$

[7]A specific representation of $J_1(q_0)$ is given in Equation (7.4) of the Appendix.

[8]The signs of $J_1(q_0)$ and $J_{11}(q_0)$ are determined in (7.4) and (7.5) of the Appendix.

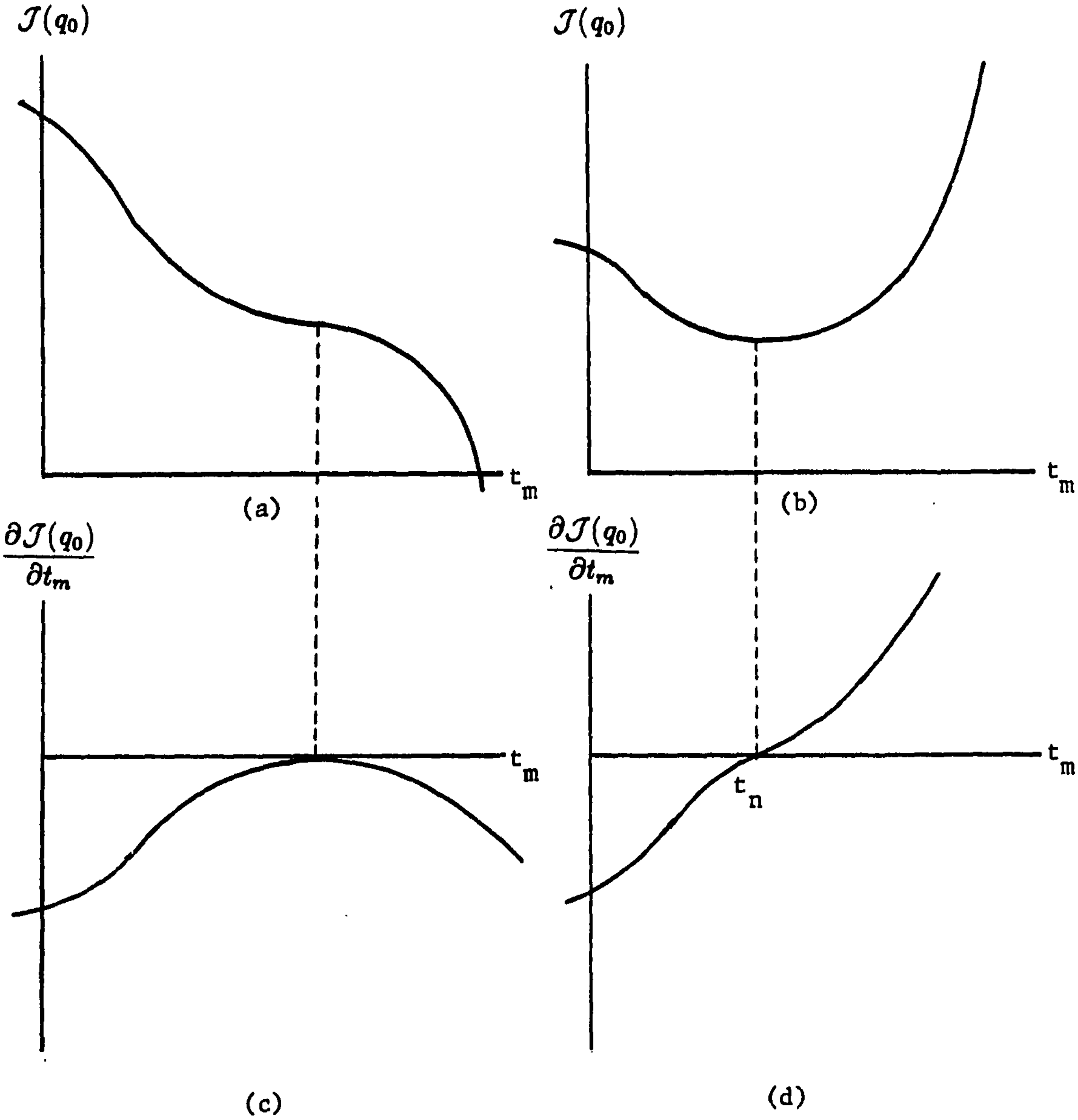

Figure 4.3: Functional form of $\mathcal{J}(q_0)$: when $q_0 > q_s^*$

The implication of (4.16) is shown in Figure 4.3, which demonstrates the relationship between the control starting point t_m and the corresponding level of $\mathcal{J}(q_0)$. What is implied by equation (4.16) is that function $\mathcal{J}(q_0)$ is decreasing at $t_m = 0$ when $q_0 > q_s^*$. Even though we know that $\partial\mathcal{J}(q_0)/\partial t_m < 0$ at $t_m = 0$, this information does not necessarily guarantee that $\mathcal{J}(q_0)$ is at maximum when $t_m = 0$. This is because we cannot exclude such a case as shown in panel (b). If we can show that function $\mathcal{J}(q_0)$ is a non-increasing function like the curve shown in panel (a), then it can fulfill the second-order sufficiency condition.

We can prove the non-increasing property of function $\mathcal{J}(q_0)$ by showing a contradiction that arises when function $\mathcal{J}(q_0)$ is assumed to be increasing for a certain positive value of t_m. We can show that the second derivative of $\mathcal{J}(q_0)$ with respect to t_m is

$$\frac{\partial^2 \mathcal{J}(q_0)}{\partial t_m^2} = -r\frac{\partial \mathcal{J}(q_0)}{\partial t_m} \tag{4.17}$$

$$-\delta e^{-(r+\delta)t_m}(\bar{q}_s - q_0)\left[D_1(q_{t_m}) + (r+\delta)J_1(q_{t_m}) - \delta(\bar{q}_s - q_0)J_{11}(q_{t_m})\right]$$

At this stage, the sign of the above equation cannot be determined with the direct information given so far. But we know that $\mathcal{J}(q_0)$ is continuous and differentiable in t_m. Suppose that $\mathcal{J}(q_0)$ is locally increasing in t_m for some q_0. Then there should exist a local minimal point at which $\partial\mathcal{J}(q_0)/\partial t_m = 0$, and $\partial\mathcal{J}^2(q_0)/\partial t_m^2 > 0$, because $\partial\mathcal{J}(q_0)/\partial t_m < 0$ at $t_m = 0$ if $q_0 < q_s^*$. A simple glance at panel (b) of Figure 4.3 tells us why such a point should exist for a local increase of $\mathcal{J}(q_0)$. Let us denote this point of time corresponding to the minimal point of $\mathcal{J}(\cdot)$ to be t_n as has been represented in panel (b) and panel (d). Substituting the first-order condition that

$\partial\mathcal{J}(q_0)/\partial t_m = 0$ at $t_m = t_n$ into equation (4.17), it will take the following specific form.

$$\frac{\partial^2\mathcal{J}(q_0)}{\partial t_m^2}|_{t_m=t_n} = -\delta e^{-(r+\delta)t_n}(\bar{q}_s - q_0) \tag{4.18}$$

$$[D_1(q_{t_n}) + (r+\delta)J_1(q_{t_n}) - \delta(\bar{q}_s - q_0)J_{11}(q_{t_n})]$$

Equation (4.15) tells us that the sign of the whole term in the bracket on the right hand side of the above equation is positive. Hence, the sign of Equation (4.18) turns out to be negative. This means that

$$\text{if } \frac{\partial\mathcal{J}(q_0)}{\partial t_m} = 0, \text{ then } \frac{\partial^2\mathcal{J}(q_0)}{\partial t_m^2} < 0,$$

which is contradiction to the assumption of a local minimal point. Hence, $\mathcal{J}(q_0)$ should always be non-increasing if $q_0 > q_s^*$ and the optimal starting point is at $t_m = 0$.

Of course, the above contradiction does not exclude the possibility of the existence of a point where $\partial\mathcal{J}(q_0)/\partial t_m = 0$, but it will always be an inflection point just like the point t_r in panel (c) of Figure 4.3. The non-strictness in the sign of $\partial\mathcal{J}(q_0)/\partial t_m$, however, does not affect the uniqueness of the optimality at $t_m = 0$ because of the strict inequality sign in (4.15). All of these statements can be summarized into the following proposition.

Proposition 7 *When the initial state (q_0) is more deteriorated than the first-best steady state (q_s^*), it is always beneficial to start the control immediately.*

Until now, the analysis in this chapter has only focused on the cost and the benefit that are directly related to the change of state in the environmental stock and the emission level. In the implementation of a control policy, a high level of administrative cost may have a significant effect in a decision making.[9] In the practice of a new control policy, a substantial amount of set-up cost will be involved at the beginning of the policy implementation as well as the operating cost, while the policy is being exercised. For the convenience of simplicity, let us assume that both the set-up cost, denoted as C_s, and the operating cost, denoted as C_o, are fixed amounts regardless of the level of control. When these administrative costs are introduced, the associated first-order condition for the optimal starting point of control will be modified into the following form.

$$\frac{\partial \mathcal{J}(q_0)}{\partial t_m} = e^{-rt_m}\left(\left[B(\bar{X}) - D(q_{t_m})\right] + \delta(\bar{q}_s - q_{t_m})J_1^c(q_{t_m}) - r\left[J^c(q_{t_m}) - C_s\right]\right), \tag{4.19}$$

$$\text{where } J^c(q_{t_m}) = \max_{X} \int_0^\infty e^{-rt}\left[B(X) - D(q_t) - C_o\right] dt.$$

We can show that an increase in any of the administrative costs justifies a further delay in the start of the control. Setting the right hand side of (4.19) to be zero and using the implicit function rule, we can show that

$$\frac{dt_m}{dC_s} = \frac{-r}{e^{-rt}\left[-D_1 - (r+\delta)J_1^c + \delta(\bar{q}_s - q_{t_m})J_{11}^c\right](\partial q_{t_m}/\partial t_m)} > 0, \tag{4.20}$$

[9]For example, Polinsky and Shavell (1982) showed how the optimal tax should be adjusted when an implementation of a Pigouvian tax requires a substantial administrative cost.

$$\frac{dt_m}{dC_o} = \frac{-1}{e^{-rt}\left[-D_1 - (r+\delta)J_1^c + \delta(\bar{q}_s - q_{t_m})J_{11}^c\right](\partial q_{t_m}/\partial t_m)} > 0, \qquad (4.21)$$

In the interpretation of the preceding results, it is important to remember that, in the formulation of the model, the controlling authority is presumed to act in a rational or impartial way: impartial as for the timing of the realizations of the cost and the benefit. In the real world, a controller's decision may not necessarily be so impartial or rational. The set-up cost actually has the same meaning as a long-term investment, not all the benefit of which is realized during the tenure of the incumbent controller.

The controller can perceive that, by deferring the control until the end of his tenure, he can save not the interest rate of long-term investment but the huge amount of cost that would otherwise be used in pure consumption for the contemporary constituents, let alone the possible repercussion of those who are potentially subject to the control. A lumpy set-up cost can provide an incentive for an administrator with finite tenancy to postpone the initiation of a control policy to his successors. In addition, as the size of the initial set-up cost rises, the tendency for the control policy to be put off further into the future would be even greater. This can explain partly the delay in the start of control on a number of negative externality problems in the real world.

5 COMPARISON OF ALTERNATIVE CONTROL MODES

5.1 Introduction

One of the most commonly perceived notions, among economists, seems to be that a price control cannot be inferior to a direct quantity control.[1] Once the target of a control is given, financial incentives can automatically achieve the target at the least cost. But, in the real world, examples of direct quantity control are more easily witnessed than the cases of price control. Economists have tried to explain the reasons for the frequent use of quantity controls with some factors other than those of economics: bureaucratic characteristics of the controlling authority, relative enforceability and simplicity of the scheme, difficulties in legal process, etc.

Buchanan and Tullock (1975) tried to explain the dominant adoption of direct regulation in the context of a public choice approach. They showed that, in a competitive industry, when the number of firms is controlled, each polluter's quasi-rent under direct quantity control cannot be lower than that under price control. This would generate a strong incentive for polluters to influence the controller to choose a quantity control even though it would be inferior to a price control from the standpoint of the efficiency of the economy as a whole. In a democratic country "a

[1] Under a limiting assumption that all the information is distributed at zero price and the administration cost is also zero, both control schemes will be equivalent on efficiency grounds but not in distributional terms.

small, concentrated, identifiable, and intensely interested pressure group may exert more influence on political choice making than the much larger majority of persons, each of whom might expect to secure benefits in the second order of smalls."[2]

On the other hand, there has been a different line of thought on the same problem. Some economists have tried to explain the cause of frequent adoption of quantity control with economic factors. Lerner (1971), for instance, noted the limitation in available information and maintained that a controlling authority should rely on the better-known information of either the marginal cost or the marginal damage curves. If the position of one curve is known better than the other and the better-known curve is more horizontal, then price control is more efficient. If the better-known one is vertical, then the quantity control would perform better.

The comparison between these two policy alternatives has come to be well established by the seminal work of Weitzman (1974). He derived the result that, if the cost and the benefit functions are randomly fluctuating, the ranking of these two policy alternatives depends on the relative slopes of the marginal cost and the marginal benefit curves.[3] For instance, when the marginal benefit curve is steeper than the marginal cost curve, a small deviation of the emission from the optimal level would lead to a disastrous result and a restrictive quantity control can reduce the possibility of severe damage. If the marginal benefit curve is flatter than the marginal cost curve, then price control is more desirable because it allows the

[2]Buchanan and Tullock (1975, p. 142).

[3]In Weitzman (1974), the explanatory variable is represented in terms of an environmental good, which is the opposite of an environmental bad, or more simply a pollution level. Therefore, an increase in the benefit or an increase in the cost in Weitzman's paper actually corresponds to a decrease in the damage or a decrease in benefit in this dissertation.

individual polluter to choose the optimal level of pollution for a relatively stable external cost.

Direct extensions of Weitzman's result have been tried by such economists as Laffont (1977), Ireland (1977), and Yohe (1978), but in most cases the changes were not substantial. Weitzman's particular methodology of comparing two second-best policy alternatives has been applied in the studies of government regulation such as Mendelsohn (1980 and 1986), Beavis and Dobbs (1987) and Browning (1987). However, all of them are in a static framework and the issue has not yet been examined in a dynamic setting.

In the field of international economics, the comparison of tariffs and quotas has been a very popular topic in the last decade, and the theoretical development has been substantial. It is a well-known theory that tariffs and quotas are equivalent in a certainty case. However, when some uncertain factors from a variety of sources are incorporated, and also when other conditions like risk attitudes of the agents are added, opposing results have been derived on the ranking of these two policy measures.[4]

In this chapter, the main issue is to compare the economic efficiencies of the two second-best policies, a tax system and a quantity restriction on a stock external problem. In the model developed below, the effects of two different sources of random factors will be considered: one random disturbance in the benefit function, and the other one in the equation of motion. As in the certainty case, policies are presumed to be exercised only with fixed one-time controls, which characterize these

[4]Some of the representative studies in this issue are Fishelson and Flatters (1975), Pelcovitz (1976), Dasgupta and Stiglitz (1977), and Young and Anderson (1980 and 1982).

controls to be second-best policies.

5.2 Formulation of the Model

In Chapter 3, it was already mentioned that there is no difference in economic performance between a quantity restriction and a Pigouvian tax under the certainty assumption. However, when it comes to uncertainty, one policy would be different from the other in their economic effects because a tax system can take the changing economic environment into account while quantity restriction cannot. Of course, the random factors in the benefit function and in the equation of motion may not be the only sources of uncertainty. In fact, the level of damage is also subject to a certain random effect. For instance, variations in weather conditions, changes in the number of pollutees and their locations are some of the factors that can affect the level of damage for a given state of the stock. However, the random factor in the damage function is not the main source of the difference in the relative economic efficiencies of tax system and quantity restriction. Rather, the uncertainty from this source might affect the economic performances of both control modes equally or at least to the same direction, such that the relative ranking of the two control policies is not affected.[5]

On the benefit side, the polluters have more specific information on their production or consumption environment. But the polluters do not have any incentives to reveal this information voluntarily unless their objectives are the same as that of the controller. In other cases, these information cannot be utilized by the controller

[5]The random factor in the equation of motion does not affect the ranking of the two policies. It will be explained in more detail in a later part of this section.

because of the complexity of institutional restrictions. In the particular case of a fixed one-time control, the regulating authority should determine the level of control at the beginning of the planning horizon. Therefore, the only kind of information available to the controller is, at most, a probability distribution of a certain random variable.

If a serially uncorrelated random factor ϵ_t whose mean is zero and variance is σ_ϵ^2 is incorporated as an additional argument of the benefit function, then it will be in the form of $B(X_t, \epsilon_t)$. In each period, the random factor ϵ_t is assumed to be known to the polluter before the production (or consumption) decision is made. However, it does not matter whether, in each period, the random factor is known to the controller *ex ante* or not since the policy variables cannot be changed throughout the subsequent periods once they are chosen at the beginning of the control.

In order to make the problem tractable, we need to change Equation (3.1) into a difference equation form with an additional argument of random factor θ_t:

$$q_t = \alpha X_{t-1} + (1-\delta)q_{t-1} + \theta_{t-1}, \text{ for } t > 1, \tag{5.1}$$

where $0 < \delta < 1, E(\theta_t) = 0, E(\theta_t^2) = \sigma_\theta^2$, and θ_t is serially uncorrelated. To make the problem simple, we are going to assume that $E(\theta_i \epsilon_j) = 0$ for all i and j. The value of δ characterizes the problem as follows: if $\delta \longrightarrow 1$, then q_t becomes a flow; if $\delta \longrightarrow 0$, then pollution activity is similar to the extraction of an exhaustible resource. By substituting recursively, the current state (q_t) can be expressed as the summation of the emissions in the past, the history of the random factor θ_i, and the initial state q_0 where all of them are discounted with $(1-\delta)$ for each period. The result is shown in Equation (5.2):

$$q_t = \alpha \sum_{i=1}^{t}(1-\delta)^{i-1}X_{t-i} + \sum_{i=1}^{t}(1-\delta)^{i-1}\theta_{t-i} + (1-\delta)^t q_0. \qquad (5.2)$$

The objective function with a fixed one-time quantity restriction is

$$EJ_Q(q_0) = \max_{X} \; E\sum_{t=0}^{\infty}(\frac{1}{1+r})^t \left[B(X,\epsilon_t) - D(q_t)\right], \qquad (5.3)$$

$$\text{subject to } q_t = \alpha X \sum_{i=1}^{t}(1-\delta)^{i-1} + \sum_{i=1}^{t}(1-\delta)^{i-1}\theta_{t-i} + (1-\delta)^t q_0.$$

In (5.3), all X's are without time index t because X is constant over time under a quantity restriction. The associated first-order condition is

$$\frac{\partial EJ_Q(q_0)}{\partial X} = E\sum_{t=0}^{\infty}(\frac{1}{1+r})^t \left[\frac{\partial B(X,\epsilon_t)}{\partial X} - \frac{dD_t}{dq_t}\frac{\partial q_t}{\partial X}\right] = 0, \qquad (5.4)$$

$$\text{subject to } q_t = \alpha X \sum_{i=1}^{t}(1-\delta)^{i-1} + \sum_{i=1}^{t}(1-\delta)^{i-1}\theta_{t-i} + (1-\delta)^t q_0.$$

Under a tax system, the objective function is

$$EJ_p(q_0) = \max_{p} \; E\sum_{t=0}^{\infty}(\frac{1}{1+r})^t \left[B(X_t,\epsilon_t) - D(q_t)\right], \qquad (5.5)$$

$$\text{subject to } q_t = \alpha \sum_{i=1}^{t}(1-\delta)^{i-1}X_{t-i} + \sum_{i=1}^{t}(1-\delta)^{i-1}\theta_{t-i} + (1-\delta)^t q_0,$$

$$\text{and } B_1(X_t,\epsilon_t) = p.$$

The second constraint in (5.5) is the polluter's behavioral equation. For a given tax rate p, the polluter will equate his marginal benefit to p in order to maximize his own benefit. A profit- (or utility-) maximizing polluter will change the level of emission as the production (or consumption) environment changes. Therefore, X_t is a function of ϵ_t conditioned on a given level of tax rate p. This is why the emission level X_t is represented as a function of a tax rate p and the random factor ϵ_t.

The first-order condition corresponding to (5.5) is

$$\frac{\partial EJ_p(q_0)}{\partial p} = E\sum_{t=0}^{\infty}(\frac{1}{1+r})^t \left[\frac{\partial B_t}{\partial X}\frac{\partial X(p,\epsilon_t)}{\partial p} - \frac{dD_t}{dq_t}\frac{\partial q_t}{\partial X}\frac{\partial X(p,\epsilon_t)}{\partial p}\right] = 0. \tag{5.6}$$

$$\text{subject to } q_t = \alpha\sum_{i=1}^{t}(1-\delta)^{i-1}X_{t-i} + \sum_{i=1}^{t}(1-\delta)^{i-1}\theta_{t-i} + (1-\delta)^t q_0,$$

$$\text{and } B_1(X_t,\epsilon_t) = p.$$

Under a quantity restriction, the emission level $\hat{X}$ is constant over time but the equation of motion involves the random factor θ_t, so q_t is only subject to the effect of random variable θ_t. However, under a tax system, the emission level X_t is affected by the random factor ϵ_t in the benefit function. The effect of ϵ_t is, in turn, transmitted to the damage function *via* the equation of motion, and the state of the stock is subject to the effects of two random variables, ϵ_t and θ_t. Such a difference in their policy impacts makes the following conjecture possible. Under price control, a pollution-generating producer (or consumer) can adjust to the randomly changing environment more freely than under a quantity restriction. But such adjustability will yield greater fluctuations in the level of pollution over time and it will be associated with more dangerous results than had the quantity restriction been the case.

Because of the effect of the random factor, one of the control modes cannot be equivalent to the other in an uncertainty case. However, we can simply ask whether the expected level of emission under the tax system is equal to the level of emission under the quantity restriction. In the certainty case, the tax system and the quantity restriction are equivalent because the common factor dX/dp in the integrand of Equation (3.16) is constant over time and it can be factored out. In the presence of the random factor in the benefit function, the common factor $\partial X/\partial p$ cannot be dropped out. In general, both $\partial X/\partial p$ and $\partial B_t/\partial X_t$ are functions of ϵ_t and the covariance between $\partial B_t/\partial X_t$ and $\partial X_t/\partial p$ is not equal to zero, so $E[(\partial B_t/\partial X_t)(\partial X_t/\partial p)]$ is not equal to $E(\partial B_t/\partial X_t)E(\partial X_t/\partial p)$.[6] Therefore, we cannot expect it to be a general result that $EX(\tilde{p}) = \hat{X}$,[7] where $X(\cdot)$ is the level of emission under tax, $\tilde{p}$ is the solution of equation (5.6), and $\hat{X}$ is the solution of equation (5.4).

Since Equations (5.4) and (5.6) can hardly be solved in their original general forms, the analyses will be conducted with an approximation technique. By taking a Taylor expansion of the benefit function to a second order[8] around a certain point, where $X_t = \hat{X}$ and $\epsilon_t = 0$, the benefit function is approximated such that

[6]If ϵ_t enters into the function of $\partial B/\partial X_t$ in an additively separable form, then $E[(\partial B_t/\partial X_t)(\partial X_t/\partial p)] = E(\partial B_t/\partial X_t)E(\partial X_t/\partial p)$.

[7]There is one exceptional case. When $\delta \longrightarrow 1$, it follows that $EX(\tilde{p}) = \hat{X}$. However, this is not a stock problem, but a static flow problem. As for this particular case, it will be explained more specifically in a later part of this section.

[8]The discussions on the justification of a second-order approximation are available in Samuelson (1970), Malcomson (1977), and Weitzman (1977).

$$B(X_t,\epsilon_t) \cong B(\hat{X},0) + B_1(\hat{X},0)(X_t-\hat{X}) + B_2(\hat{X},0)\epsilon_t + \tag{5.7}$$

$$\frac{B_{11}(\hat{X},0)(X_t-\hat{X})^2}{2} + \frac{B_{22}(\hat{X},0)\epsilon_t^2}{2} + B_{12}(\hat{X},0)(X_t-\hat{X})\epsilon_t.$$

The second order approximation of the damage function around a certain point $\hat{q}_s$ is

$$D(q_t) \cong D(\hat{q}_s) + D_1(q_t-\hat{q}_s) + \frac{D_{11}}{2}(q_t-\hat{q}_s)^2. \tag{5.8}$$

Rearranging the terms in (5.7) and (5.8), and dropping the constant terms,[9] we would end up with simple quadratic forms of benefit and damage functions as given in (5.9) and (5.10).

$$B_t \equiv B_1X_t + \frac{B_{11}}{2}X_t^2 + B_{12}X_t\epsilon_t + \frac{B_{22}}{2}\epsilon_t^2 \tag{5.9}$$

$$D_t \equiv D_1q_t + \frac{D_{11}}{2}q_t^2. \tag{5.10}$$

A great advantage of using a quadratic benefit function that is based on a second-order approximation is that the random variable ϵ_t enters the function either in an additive or linearly multiplicative form with respect to the other variable X_t. In fact, these quadratic functions are qualitatively equivalent to those functions of second-order approximation in the sense that their solutions are in the same form. As compared with the original second-order approximation forms in (5.7) and (5.8), the simple quadratic functions in (5.9) and (5.10) are easy to handle, so these quadratic benefit and damage functions will be used in the following discussion.

[9]Dropping the constant terms does not affect the solutions at all.

If the controller imposes an emission charge at the rate of p for a unit of X, then the polluter's maximization will end up with the following decision:

$$p = \frac{\partial B(X_t, \epsilon_t)}{\partial X_t} = B_1 + B_{11}X_t + B_{12}\epsilon_t. \tag{5.11}$$

This equation can be interpreted as the inverse demand function of the emission X_t, where the tax rate p is perceived as a given market price of the emission. Here, the random factor remains in the demand function of X_t in additive form and the emission level under a Pigouvian tax can be decomposed into a constant term and a random component.

$$X_t = \frac{1}{B_{11}}(p - B_{11}) - \left(\frac{B_{12}}{B_{11}}\right)\epsilon_t = X_e - \left(\frac{B_{12}}{B_{11}}\right)\epsilon_t, \tag{5.12}$$

$$\text{where } X_e = E(X_t) = \frac{1}{b_{11}}(p - B_1).$$

Since X_e is a linear function of p, a specific value of X_e automatically determines a unique value of p. This implies that we can reformulate the objective function in (5.5) in such a way that X_e is chosen to be the decision variable, instead of p. By substituting (5.12) into (5.2) and with a proper change in its time index, the solution of q_t under a Pigouvian tax is

$$q_t = \alpha X_e \sum_{i=1}^{t}(1-\delta)^{i-1} + \sum_{i=1}^{t}(1-\delta)^{i-1}\left[\theta_{t-i} - \left(\alpha\frac{B_{12}}{B_{11}}\right)\epsilon_{t-i}\right] + (1-\delta)^t q_0. \tag{5.13}$$

Equations (5.9), (5.10), (5.12) and (5.13) are sufficient to determine the optimal X_e, and specific form of the objective function for a Pigouvian tax system is given at (7.15) in the Appendix.

Under a quantity restriction, the recursive solution of q_t does not contain ϵ_i, so it will assume the following form:

$$q_t = \alpha X \sum_{i=1}^{t}(1-\delta)^{i-1} + \sum_{i=1}^{t}(1-\delta)^{i-1}\theta_{t-i} + (1-\delta)^t q_0. \tag{5.14}$$

By substituting (5.9), (5.10), and (5.14) into (5.3) and simplifying it, the specific form of the objective function for a one-time quantity restriction is derived, which is given in Equation (7.10) of the Appendix. Equations of (7.10) and (7.15) are the basis for the comparison of the economic efficiencies between the two control policies in the next section.

5.3 Welfare Comparison of Tax System and Quantity Restriction

A comparison of the two equations (7.10) and (7.15) tells us that all the coefficients of the decision variables are exactly the same in both equations. This implies that the quantity restriction and the tax system will end up with the same first order condition. As a consequence, the optimal level of emission under the quantity restriction coincides with that of the Pigouvian tax system, *i.e.*, $EX(p, \epsilon_t) = X_e = \hat{X}$. This result is an important condition in the welfare comparison of the two control modes because the equivalence of X_e and $\hat{X}$ also guarantees that the optimal mean values of the stock variables $\tilde{q}_t$ and $\hat{q}_t$ are equal under the two different policies ($E\tilde{q}_t = E\hat{q}_t$), throughout the whole planning horizon.

In the preceding section of this chapter, it was indicated that the optimally restricted quantity emission $\hat{X}$ and the expected level of emission level under a Pigouvian tax $\tilde{X}_e$ are not generally equal. The result that $\hat{X} = \tilde{X}_e$ is rather the particular consequence from the quadratic benefit and damage functions. However, it is important to notice that both of the quadratic functions are not arbitrarily chosen, but they are based on the approximations of the general functional forms. The fact that the non-equivalence of $\tilde{X}_e$ and $\hat{X}$ cannot be captured with a second-order approximation allows us to make the following conjecture: the difference in $\tilde{X}_e$ and $\hat{X}$ may not be a major source of the welfare difference between tax system and quantity restriction if the benefit and damage function conform to a certain regularity.[10]

Let us define the relative advantage of a tax system over a quantity restriction (Δ), such that

$$\Delta = EJ_p - EJ_Q \tag{5.15}$$

We know that $\hat{X} = \tilde{X}_e$. Incorporating this equivalence result into (7.10) and (7.15), and subtracting (7.10) from (7.15), the relative advantage of the tax system as compared to the quantity restriction is derived:

$$\Delta = -\frac{\sigma_e^2}{2}\left(\frac{B_{12}}{B_{11}}\right)^2 \left(\frac{1+r}{r}\right)\left[B_{11} + \frac{\alpha^2 D_{11}}{(1+r)-(1-\delta)^2}\right]. \tag{5.16}$$

There are several things to indicate in relation to the above result. The first one is that the random factor in the equation of motion (θ_t) does not appear in (5.16), so θ_t does not make any difference in the economic performances of the two policies. The equation of motion is the information about the change of the environmental stock over time, and the polluters do not have any incentive to pay attention to it.

[10]If the benefit function and the damage functions are monotone and either concave or convex, then Taylor series up to second orders can approximate the original functions satisfactorily.

Actually, the policy variables, the tax rate or the restricted quantity, are supposed to send a signal to the polluters based on the information about the damage side. However, both policy variables are in a fixed state and these two policy variables *equally* do not have the ability to consider the effect of any random change in the damage side. The same should have been true for the possible random effect in the damage function. For example, if the damage function had been specified in such a form as $D(q_t, \omega_t)$, where ω_t is a random factor, then any moment of ω_t would not have appeared in the final comparison result given in (5.16).

The second thing to indicate is that the slopes of marginal benefit and marginal damage functions have an important role in the determination of the ranking of economic performances of the two policies. *Ceteris paribus*, the greater the slope of the marginal benefit function, the more preferable the tax system is; the greater the slope of the marginal damage function, the more advantageous the quantity restriction is. This result is basically the same as that of the static case in Weitzman (1974). Since this is an important aspect, a more specific analysis is required.

For the convenience of analysis, the right hand side terms in (5.16) are rearranged in the following equation.

$$\Delta = -\left(\frac{1+r}{r}\right)\left[\frac{\sigma_\epsilon^2}{2}\left(\frac{B_{12}^2}{B_{11}}\right) + \frac{\sigma_\epsilon^2}{2}\left(\alpha\frac{B_{12}}{B_{11}}\right)^2 \frac{\alpha^2 D_{11}}{(1+r)-(1-\delta)^2}\right] \tag{5.17}$$

In (5.17), the common factor $(1+r)/r$ is the result of the summation from $t = 0$ to infinity. The two terms inside the bracket represent the relative disadvantage[11] of the tax system for each period. A simplifying assumption on the probability distribution of the random variable will make the explanation of these terms convenient.

[11]This is because the right hand side of (5.17) is preceded by a negative sign.

Let us assume that the random variable ϵ_t exhibits Bernoulli distribution with equal probability for both outcomes. Because of the assumption that $E(\epsilon_t) = 0$, the two outcomes of Bernoulli trial, denoted as ϵ_1 and ϵ_2, need to be symmetric with respect to the origin, *i.e.*, $\epsilon_1 = -\epsilon_2$.

The first term in the bracket of (5.17) is visualized with the simple diagram of Fig. 5.1, which is based on the inverse demand function given in (5.11). Among the three parallel marginal benefit curves in the Figure, the one in the middle represents the mean of the marginal benefit. We know that the expected value of emission under optimal tax ($\tilde{X}_e$) is equal to the optimally restricted quantity $\hat{X}$. This is why the optimal tax rate $\tilde{p}$ is associated with the optimal quantity $\hat{X}$ with respect to the middle line.

If $\epsilon_t = \epsilon_1 > 0$, then the right-hand side curve represents the true marginal benefit. In this case, the amount of emission under the optimal tax will be determined at $\tilde{X}_1$. Since the emission under quantity restriction is at the constant level of $\hat{X}$, the relative gain under the tax system is represented by the area of the trapezoid $ab\tilde{X}_1\hat{X}$ which is the sum of the triangle abe and the quadrangle $eb\tilde{X}_1\hat{X}$.[12] If this is the case, then, the relative gain of the tax system is expressed as the following equation:

$$B(\tilde{X}_1, \epsilon_1) - B(\hat{X}, \epsilon_1) = -\frac{B_1 B_{12} \epsilon_1}{B_{11}} - \frac{B_{12} \epsilon_1^2}{B_{11}}. \tag{5.18}$$

When $\epsilon_t = \epsilon_2 < 0$, then the trapezoid $cd\hat{X}\tilde{X}_2$ is the relative loss under the tax system. This area can also be calculated by taking the difference of the quadrangle

[12]It is true the benefit corresponding to this area will be absorbed as tax by the controlling authority, but the distributional aspect is not an issue in the comparison.

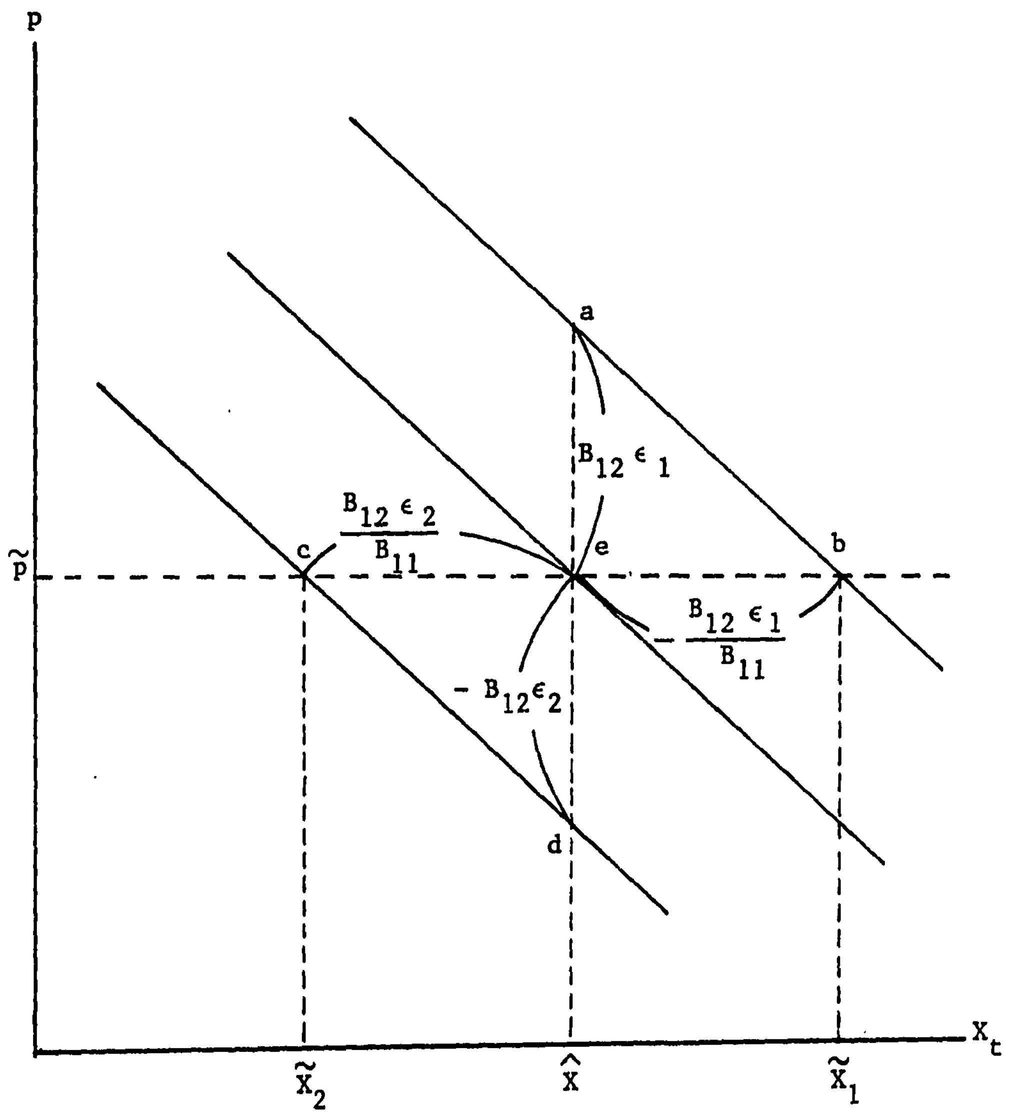

Figure 5.1: The effects of tax and quantity restriction: on the benefit side

$ce\hat{X}\tilde{X}_2$ and the triangle ced. Since the area represents the relative loss that has been incurred under the tax system, the relative gain should be represented by the negative value of this area. That is:

$$B(\tilde{X}_2, \epsilon_2) - B(\hat{X}, \epsilon_2) = -\frac{B_1 B_{12} \epsilon_2}{B_{11}} - \frac{B_{12} \epsilon_2^2}{B_{11}}. \tag{5.19}$$

Comparison of the two equations will tell us that, regardless of the sign of ϵ_t, the difference in the benefit can be expressed in the same form of equation. When we take expectations of any of the above two equations, the first term on the right hand side drops out and only the last term remains. This is only a one-period result. The total relative advantage of the tax system on an infinite time horizon is the summation of the relative advantage of each period, which can be done by simply multiplying the common factor $(1+r)/r$ that remains outside of the bracket in Equation (5.17).

The same type of graphical representation is possible for the damage side. This is shown in Figure 5.2. Since the line in the figure represents the marginal damage for a given level of q_t, the total damage corresponds to the area under the marginal damage curve to the left hand side of a given level of q_t. When $\epsilon_t = \epsilon_1$, the difference in the level of emission between the two policies is $-(B_{12}/B_{11})\epsilon_1$, and such a difference in the level of emission is transmitted into the difference in the states of stocks in the following period by equation of motion. It is assumed that only a fraction of the emission is accumulated in the stock, so the difference in the level of q_t in the following period is $-(\alpha B_{12}/B_{11})\epsilon_1$, where α represents the fraction of emission that is undissipated. Consequently, the additional damage associated with tax system corresponds to the area of the trapezoid $ac\tilde{q}_1\hat{q}_t$. By an analogy,

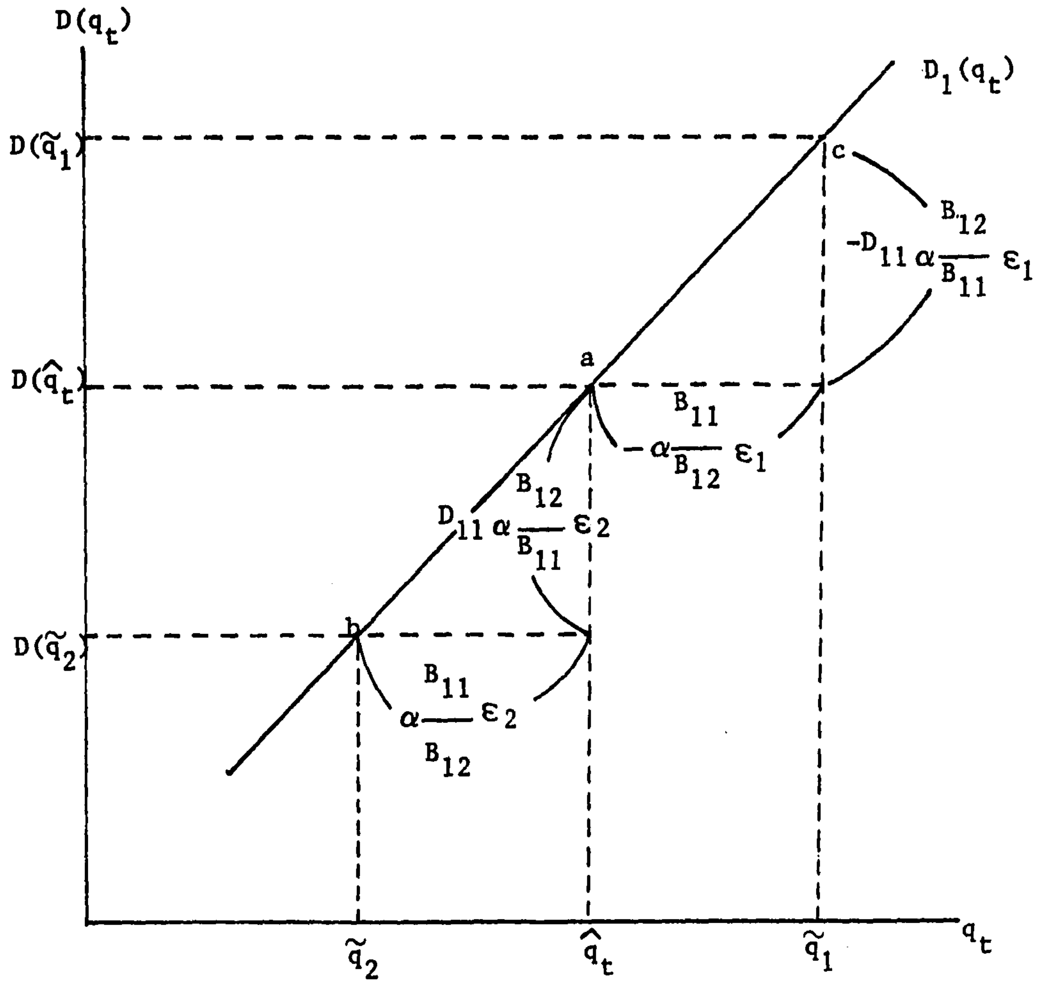

Figure 5.2: Difference in the damage between tax and quantity restriction

when $\epsilon_t = \epsilon_2$, the tax system would be associated with a lower level of damage, which is represented by the area of the trapezoid $ba\hat{q}_t\tilde{q}_2$. Since the probability of each outcome is 1/2, the expected value of the difference is (area $ac\tilde{q}_2\hat{q}_t$ - area $ba\hat{q}_t\tilde{q}_1$)/2, which is equal to $D_{11}\sigma_\epsilon^2(\alpha B_{12}/B_{11})^2/2$. If we assume that the emission is discharged at the period of $t = 0$, this expected value is the additional damage under the tax system in period $t = 1$, and the corresponding present value is $D_{11}\sigma_\epsilon^2(\alpha B_{12}/B_{11})^2/2(1+r)$.

Since any difference in the state of the stock in a certain period will be transmitted into the following period, the discharge in period 0 will affect the state of the stock in period 2, too. The expected value of the difference in the damage in period 2 is $(1-\delta)^2 D_{11}\sigma_\epsilon^2(\alpha B_{12}/B_{11})^2/2$, and its present value is $(1-\delta)^2 D_{11}\sigma_\epsilon^2(\alpha B_{12}/B_{11})^2/2(1+r)^2$. This series will continue in all the subsequent periods and the summation of this series is equal to the second term in the bracket of (5.17).

In calculating the total effect in the damage, what we have to notice is the fact that the above series is the consequence of the expected difference in emission only for one period. Since the basic assumption of one-time control is that the policy chosen at the beginning of the planning horizon will be in place throughout the subsequent periods, continuation of one policy scheme will generate the same amount of difference in the emission levels for every period in the future. Therefore, the total difference in the expected damage is calculated by summing up all the differences in the expected damages associated with the difference in the emission of every future time period, with a proper discount for each period. The result of this summation is a simple multiplication of the common factor outside the bracket, $(1+r)/r$, and the second term inside the bracket. By subtracting the total relative disadvantage on the damage side from the total relative advantage on the benefit side, the final form of the relative advantage of the tax system compared with the quantity restriction will be equal to (5.17).

In Equation (5.17), the discount rate r can also affect the controller's decision. The benefit is realized in the same period when the emission is discharged, but the resulting damage occurs in the future. A higher discount rate means assigning greater value to the present benefit and less weight to the worry about future damage. Therefore, a higher discount rate is likely to lead to the conclusion that a tax system is more attractive because it is associated with higher current benefits and potentially more risky outcomes in the future.

The amelioration rate δ has a critical role in the determination of the degree of interconnection between periods. A higher value in δ means that a decision made at the present moment will affect the future state with lower degree. As a consequence, the amelioration rate δ has a negative relationship with the difference in the variances of $\tilde{q}_t$ and $\hat{q}_t$ in the future. The difference in the variances can be calculated by using (5.13) and (5.14) and this difference gets greater over time, asymptotically approaching a certain value, which will be shown in the following.

Let us define the difference in the variances of the two policy measures in such a way that

$$\Delta\sigma^2_{q_t} \equiv \mathrm{Var}(\tilde{q}_t) - \mathrm{Var}(\hat{q}_t) \tag{5.20}$$

$$= E\left[-\sum_{i=1}^{t}(1-\delta)^{i-1}\left(\alpha\frac{B_{12}}{B_{11}}\right)\epsilon_{t-i}\right]^2 = \frac{1-(1-\delta)^{2t}}{1-(1-\delta)^2}\left(\alpha\frac{B_{12}}{B_{11}}\right)^2\sigma^2_\epsilon.$$

For convenience of calculation, but without losing any generality, we can pick one future time period T, where $T \longrightarrow \infty$, and show the relationship between the amelioration rate δ and the difference in variances, $\Delta\sigma^2_{q_T}$:

$$\Delta\sigma^2_{q_T} = \frac{1}{\delta(2-\delta)}\left(\alpha\frac{B_{12}}{B_{11}}\right)^2 \sigma^2_\epsilon, \tag{5.21}$$

$$\frac{\partial\Delta\sigma^2_{q_T}}{\partial\delta} = -\frac{2(1-\delta)}{[\delta(2-\delta)]^2}\left(\alpha\frac{B_{11}}{B_{12}}\right)^2 \sigma^2_\epsilon < 0,$$

$$\frac{\partial^2\Delta\sigma^2_{q_T}}{\partial\delta^2} = \frac{2\delta(2-\delta)+8(1-\delta)^2}{[\delta(2-\delta)]^3}\left(\alpha\frac{B_{12}}{B_{11}}\right)^2 \sigma^2_\epsilon > 0.$$

Figure 5.3 shows the relationship between δ and $\Delta\sigma^2_{q_T}$. This relationship tells us that for certain externality problems which would persist for a protracted period, those policy measures which are based on financial incentives are not desirable on purely economic grounds. With a Pigouvian tax system there can be some economic gain in the current period, but it will generate a high degree of uncertainty in the level of the future damage. Therefore, in this case, direct quantity regulation is more desirable, *ceteris paribus.*

In Section B of this chapter, it was indicated that our model can be applied to the case of flow externality. When $\delta \longrightarrow 1$, equation (5. 16) boils down to the following form:

$$\Delta = -\frac{\sigma^2_\epsilon}{2}\left(\frac{B_{12}}{B_{11}}\right)^2 \left(\frac{1+r}{r}\right)\left[B_{11} + \frac{\alpha^2 D_{11}}{1+r}\right], \text{ when } \delta \longrightarrow 1. \tag{5.22}$$

This equation is basically the same result as that of Weitzman's original work.[13]

[13]Equation (20) in Weitzman (1974, p. 484).

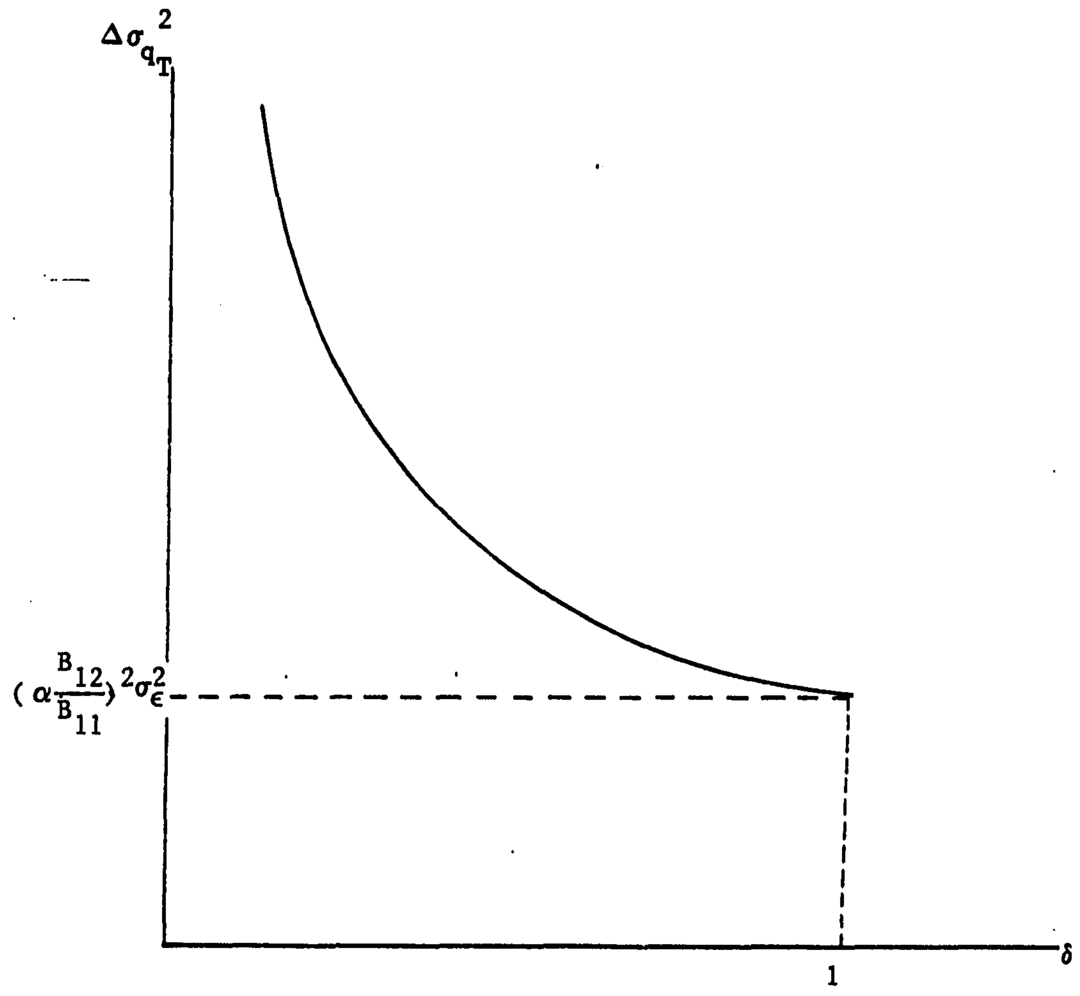

Figure 5.3: The effect of dispersion rate δ on the variance of damage

Therefore, it can be maintained that Weitzman's original model in a static framework is only a particular case of the dynamic framework developed in this chapter.

6 SUMMARY

In modern economics, the issues associated with externality are so diverse that there are still numerous questions that has not been answered. This indicates the possibility or desirability of the further exploration in this area and the high applicability of any economic theory aimed at the externality problem. As has been indicated by the title, the main issues of this dissertation are those problems that are associated with the control of stock externality under the constraint that the policy variables, such as a Pigouvian tax rate or a restricted quantity cannot be changed frequently. These are the issues that have been examined in Chapters 3, 4, and 5.

Chapter 2 provides a general review on the theory of externality and the possible directions of future studies in this area. In general, we can say that an externality is present whenever one agent's economic decision can directly affect the welfare state of other's without any compensation (or bribery) or consent from the affected party. It is important to note that the concept of externality relies on a market economy where such a non-market interaction is considered to be an exceptional case. Consequently, an externality can be conceived as an absence of market. This is why the externality is classified to be a form of market failure.

The interpretation of externality as an absence of market allows us to point out the sources of externality with ease because we only need to indicate the causes of why the market has not been developed. Several factors can be listed as the potential sources of the non-existence of a market: difficulties in defining property rights, market operating costs, and limited number of buyers and sellers. Even though there are some obstacles in developing voluntary markets, these difficulties may not necessarily justify direct interventions of a government. We can indicated that removing the intertemporal externality in the set-up and operation of a market, or creating an economic condition conformable to a direct voluntary transaction, are possible solutions to an externality. But more frequently discussed policy measures are a policy scheme that is based on financial incentives, such as tax and a subsidy, or direct regulations. Direct regulations are important control measures as far as administrative convenience is concerned and the effect is relatively clear and immediate. However, from the pure economic point of view, these are often naive and inefficient.

As the deficiencies of the existing literature in the area of externality, several points have been indicated: the problem of stock externality which is the issue of this dissertation, a general equilibrium approach, the consideration of locational factors, and the incentive scheme for the revealment of correct information. All of them are difficult problems, but they are still very important for the solution of real world problems.

The general purpose of Chapter 3 is to provide the answers to the following questions: whether a rigid control variable that cannot be changed for a long period of time can still approximate the optimal path? How the economic decisions and results associated with these fixed policy measures will differ from those of the optimal control? In the case of the first-best policy, an optimal control on a stock externality requires for the control variables to be adjusted every moment of decision. But in the real world, such a continuous change in the level of control is never possible. In order to describe such an institutional constraint of the real world, the formulation of a fixed one-time control policy has been introduced. As a justification of assuming a perfectly rigid control policy, it has been maintained that: a positive rate of discounting factor in the formulation of the one-time control will give a greater weight on the effect in the current period or those in the near future at the expense of those far in the futures. In fact, there will not be any great difference in the controller's current decision making whether a policy can be changed far in the future or completely fixed forever.

The important results of Chapter 3 can be described by the relative position of the steady state under one-time control with respect to that of the first-best policy and the difference of the levels of emissions of these two policies. If the initial state of the environment is more (less) deteriorated than the first-best optimal state, then the steady-state of the stock under the second-best control will end up with a less (more) deteriorated than the first-best steady state which is independent of the given initial state. With regard to the difference in the emission levels, it is proved that the controlled emission level under a one-time control falls into the interval whose bounds are determined by the first-best optimal emission levels.

Chapter 4 examines the issue of optimal starting point of fixed one-time control. The relevance of the issue is based on the frequent observation in the real world, where an unduly long delay in the control of stock externality raises a severe social problem. First, a general rule has been formulated for the decision of optimal starting point. And then it is proved that an earlier start of control may not necessarily be desirable in the problem of stock externality if policy variable cannot be changed freely.

Another important result provided Chapter 4 is that the immediate start of a control is always desirable if the current state of the environmental stock is more deteriorated than its first-best steady state. As a related issue, it is shown that higher levels of the administrative costs, such as the cost involved in setting up the initial implementation of a control policy or the expenditure required for the exercise of the policy, will make the further delay in the start of a control more desirable. This, probably, is only a part of the explanation of the reality. As an additional description of the real world, a myopically oriented controller has a strong incentive to defer the start of a control on externality if the implementation of a control policy requires a great amount of set-up cost. The controller can perceive that, by deferring the control until the end of his tenure, he can save not the interest rate of the set-up cost but the huge amount of cost itself that would otherwise be used in the pure consumption for the contemporary constituents, let alone the possible repercussion of those who are potentially subject to the control.

Chapter 5 examines the relative performance of two representative control modes, a Pigouvian tax system and a quantity restriction. Under a deterministic setting, these two control modes are equivalent in the fact that both would end up with the same result as far as the economic efficiency is concerned. When it comes to the case of an uncertainty, they will result in different policy effects. In general, a policy scheme based on a financial incentive will be associated with a higher efficiency on the benefit side. This is because, under a price control, a pollution-generating producer (or consumer) can adjust to the randomly changing environment more freely than under a quantity restriction. But such an adjustability will yield greater fluctuations of the level of pollution over time and it will be associated with more dangerous results than the quantity restriction had been the case.

The result derived in Chapter 5 shows that the random factor in the benefit function is the only relevant source of uncertainty that affects the relative economic performance of the two policy schemes. But this random factor does not reverse the ranking of the two policies. It only affects the magnitude of the difference of the welfare states under the two policies. Since there is a time lag between the time of the realization of the benefit from emission and the resulting damage, a higher discount rate make the tax system more attractive and *vice versa.* The amelioration (or decay) rate is also an important factor that affects the economic ranking of those two policies. When the amelioration rate is low, the effect of the random fluctuation in the emission will be transmitted to the state of the environment in the future to a greater extent, which will result in a potentially more dangerous effect under the tax system. In other words, the higher the amelioration rate, the better the tax system, *ceteris paribus.*

7 APPENDIX

7.1 Section A

Equation (3.18) can be rewritten as

$$\frac{dB(\hat{X})/d\hat{X}}{r} = \int_0^\infty \left[e^{-rt} \frac{dD(\hat{q}_t)}{d\hat{q}_t} \frac{\alpha}{\delta} \left(1 - e^{-\delta t}\right) \right] dt, \tag{7.1}$$

Since $\hat{X}$, we know that:

$$\frac{\partial \hat{q}_t}{\partial \hat{X}} = \frac{\alpha}{\delta} \left[1 - e^{-\delta t}\right].$$

Take a total derivative of the above equation with respect to q_0 and $\hat{X}$ in order to determine the sign of $d\hat{X}/dq_0$, then

$$\left[\frac{d^2B(\hat{X})/d\hat{X}^2}{r}\right] d\hat{X} = \left[\int_0^\infty \left(\frac{\alpha}{\delta}(1 - e^{-\delta t})e^{-\delta t} \frac{\partial^2 D(\hat{q}_t)}{\partial \hat{q}_t^2} e^{-rt}\right)\right] dq_0 \tag{7.2}$$

$$+ \left[\int_0^\infty \left(\frac{\alpha}{\delta}(1 - e^{-\delta t})\right)^2 \frac{\partial^2 D(\hat{q}_t)}{\partial \hat{q}_t^2} e^{-rt} dt\right] d\hat{X},$$

which is rearranged into

$$\frac{d\hat{X}}{dq_0} = \frac{\int_0^\infty \left[\frac{\alpha}{\delta}(1 - e^{-\delta t})e^{-\delta t} \frac{\partial^2 D(\hat{q}_t)}{\partial \hat{q}_t^2} e^{-rt}\right] dt}{\frac{d^2B(\hat{X})/d\hat{X}^2}{r} + \int_0^\infty \left\{\left[\frac{\alpha}{\delta}(1 - e^{-\delta t})\right]^2 \frac{\partial^2 D(\hat{q}_t)}{\partial \hat{q}_t^2} e^{-rt}\right\} dt} < 0, \tag{7.3}$$

where all the terms of both of the integrands in the numerator and the denominator are positive, while the fist term in the denominator is negative.

7.2 Section B

$$\frac{dJ(q_0)}{dq_0} = \int_0^\infty \left[\frac{dB(\hat{X})}{d\hat{X}} - \frac{dD}{d\hat{q}_t}\frac{\partial \hat{X}}{\partial \hat{q}_0}\right] dt - \int_0^\infty e^{-rt}\frac{dD}{d\hat{q}_t}\frac{\partial \hat{q}_t}{\partial q_0} dt = \tag{7.4}$$

$$-\int_0^\infty \frac{dD}{d\hat{q}_t}\frac{\partial q_t}{\partial q_0} dt = -\int_0^\infty e^{-(r+\delta)t} D_1(\hat{q}_t) dt < 0,$$

by envelope theorem.

$$\frac{d^2 J(q_0)}{dq_0^2} = -\int_0^\infty e^{-rt}\left[D_{11}(\frac{\partial \hat{q}_t}{\partial q_0})^2 + D_1 \frac{\partial^2 \hat{q}_t}{\partial \hat{q}_0^2}\right] dt \tag{7.5}$$

$$= -\int_0^\infty e^{-rt} D_{11}(\frac{\partial \hat{q}_t}{\partial q_0})^2 dt < 0, \text{ since } \frac{\partial^2 q_t}{\partial q_0^2} = 0.$$

7.3 Section C

Substituting (5.9) and (5.10) into (5.3), then the objective function of the quantity restriction changes into the following form:

$$EJ_Q(q_0) = \max_X E\sum_{t=0}^{\infty}(\frac{1+r}{r})^t[B_t - D_t] \tag{7.6}$$

$$= \max_X \left\{\frac{1}{1+r}\left[B_1 X + \frac{B_{11}}{2}X^2 + \frac{B_{22}}{2}\sigma_\epsilon^2\right]\right.$$

$$\left. - \left[D_1 q_0 + \frac{D_{11}}{2}q_0^2\right] - E\sum_{t=1}^{\infty}\left[D_1(\frac{1}{1+r})^t q_t + \frac{D_{11}}{2}(\frac{1}{1+r}{}^t q_t^2\right]\right\}.$$

Equation (5.14) can be changed into the following form:

$$q_t = \frac{\alpha X}{\delta} + (1-\delta)^t(q_0 - \frac{\alpha}{\delta}) + \sum_{i=1}^{t}(1-\delta)^{i-1}\theta_{t-i}. \tag{7.7}$$

$$\text{Therefore } Eq_t = \frac{\alpha X}{\delta} + (1-\delta)^t(q_0 - \frac{\alpha X}{\delta}) \tag{7.8}$$

and

$$Eq_t^2 = (\frac{\alpha X}{\delta})^2 - 2(1-\delta)^t\frac{\alpha X}{\delta}(q_0 - \frac{\alpha X}{\delta}) + (1-\delta)^{2t}(q_0 - \frac{\alpha X}{\delta})^2 + \frac{1-(1-\delta)^{2t}}{1-(1-\delta)^2}\sigma_\theta^2 \tag{7.9}$$

Substituting (7.8) and (7.9) into (7.6) and solving it,

$$EJ_Q(q_0) = \max_X \left\{\frac{1+r}{r}\left(B_1 X + \frac{B_{11}}{2}X^2 + \frac{B_{22}}{2}\sigma_\epsilon^2\right)\right. \tag{7.10}$$

$$-\left[D_1 q_0 + \frac{D_{11}}{2}q_0^2\right] - D_1\left[\frac{\alpha X}{\delta r} + (q_0 - \frac{\alpha X}{\delta})(\frac{1}{r+\delta})\right]$$

$$-\frac{D_{11}}{2}\left[(\frac{\alpha X}{\delta})^2\frac{1}{r} + \frac{\alpha X}{\delta}(q_0 - \frac{\alpha X}{\delta})(\frac{1}{r+\delta}) + (q_0 - \frac{\alpha X}{\delta})^2(\frac{1}{(1+r)-(1-\delta)^2})\right.$$

$$\left.\left. + \frac{\sigma_\theta^2}{1-(1-\delta)^2}(\frac{1}{r} - \frac{(1-\delta)^2}{(1+r)-(1-\delta)^2})\right]\right\}.$$

7.4 Section D

Substitute (5.9), (5.10), and (5.12) into (5.5), then

$$EJ_p(q_0) = EJ_{X_e}(q_0) = \max_{X_e} E\sum_{t=0}^{\infty}(\frac{1}{1+r})^t[B_t - D_t] \tag{7.11}$$

$$= \max_{X_e} \left\{(\frac{1+r}{r})\left[B_1 X_e + \frac{B_{11}}{2}X_e^2 + \frac{B_{22}}{2}\sigma_\epsilon^2\right] - \left[D_1 q_0 + \frac{D_{11}}{2}q_0^2\right]\right.$$

$$\left. = -E\sum_{t=1}^{\infty}\left[D_1(\frac{1}{1+r})^t q_t + \frac{D_{11}}{2}(\frac{1}{1+r})^t q_t^2\right]\right\}$$

Equation (5.13) can be changed into the following form:

$$q_t = \frac{\alpha X_e}{\delta} + (1-\delta)^t(q_0 - \frac{\alpha X_e}{\delta}) + \sum_{i=1}^{t}(1-\delta)^{i-1}\left[\theta_{t-i} + (\alpha\frac{B_{12}}{B_{11}})\epsilon_{t-i}\right]. \tag{7.12}$$

$$\text{Hence, } Eq_t = \frac{\alpha X_e}{\delta} + (1-\delta)^t(q_0 - \frac{\alpha X_e}{\delta}), \tag{7.13}$$

$$\text{and } Eq_t^2 = (\frac{\alpha X_e}{\delta})^2 - 2(1-\delta)^t\frac{\alpha X_e}{\delta}(q_0 - \frac{\alpha X_e}{\delta}) \tag{7.14}$$

$$+(1-\delta)^{2t}(q_0 - \frac{\alpha X_e}{\delta})^2 + \frac{1-(1-\delta)^{2t}}{1-(1-\delta)^2}\left[\sigma_\theta^2 + (\alpha\frac{B_{12}}{B_{11}})\sigma_\epsilon^2\right].$$

Substituting (7.13) and (7.14) into (7.11) and solving,

$$EJ_{X_e}(q_0) = \max_{X_e} \left\{ \frac{1+r}{r} \left[B_1 X_e + \frac{B_{11}}{2} X_e^2 + \frac{\sigma_\epsilon^2}{2} \left(B_{22} - \frac{B_{12}^2}{B_{11}} \right) \right] \right. \tag{7.15}$$

$$- \left[D_1 q_0 + \frac{D_{11}}{2} q_0^2 \right] - D_1 \left[\frac{\alpha X_e}{\delta r} + (q_0 - \frac{\alpha X_e}{\delta})(\frac{1}{r+\delta}) \right]$$

$$-\frac{D_{11}}{2} \left[(\frac{\alpha X_e}{\delta})^2 \frac{1}{r} + \frac{\alpha X_e}{\delta}(q_0 - \frac{\alpha X_e}{\delta})(\frac{1}{r+\delta}) + (q_0 - \frac{\alpha X_e}{\delta})^2 \left(\frac{1}{(1+r)-(1-\delta)^2} \right) \right.$$

$$\left. \left. + \frac{1}{1-(1-\delta)^2} \left(\sigma_\theta^2 + (\alpha \frac{B_{12}}{B_{11}})^2 \sigma_\epsilon^2 \right) \left(\frac{1}{r} - \frac{(1-\delta)^2}{(1+r)-(1-\delta)^2} \right) \right] \right\}.$$

8 BIBLIOGRAPHY

Aivazian, V. A., and J. L. Callen. 1981. The Coase theorem and the empty core. *Journal of Law and Economics* 24:175-181.

Arrow, K. J. 1970. The organization of economic activity: issues pertinent to the choice of market versus non-market allocation. Pages 59-73 *in* Haveman, R. H. and Margolis, J., eds. *Public Expenditures and Policy Analysis.* Markham, Chicago.

Bator, F. M. 1958. The anatomy of market failure. *Quarterly Journal of Economics* 72:351-379.

Baumol, W. J. 1972. On taxation and the control of externalities. *American Economic Review* 62:307-322.

Baumol, W. J., and W. E. Oates. 1988. *The Theory of Environmental Policy.* 2nd ed. Prentice-Hall, Inc., Englewood Cliffs, New Jersey.

Beavis, B., and I. Dobbs. 1987. Firm behaviour under regulatory control of stochastic environmental wastes by probabilistic constraints. *Journal of Environmental Economics and Management* 14:112-127.

Bramhall, D. F., and E. S. Mills. 1966. A note on the asymmetry between fees and payments. *Water Resources Research* 2:355-364.

Brito, D. L. 1972. A dynamic model of an armaments race. *International Economic Review* 13:359-375.

Brito, D. L., and M. D. Intriligator. 1987. Stock externalities, Pigouvian taxation and dynamic stability. *Journal of Public Economics* 33:59-72.

Browning, M. 1987. Prices *vs.* quantities *vs. laissez-faire. Review of Economic*

Studies 54:691-694.

Buchanan, J. M. 1969. External diseconomies, corrective taxes and market structure. *American Economic Review* 59:174- 177.

Buchanan, J. M., and W. C. Stubblebine. 1962. Externality. *Economica* 29:371-384.

Buchanan, J. M., and G. Tullock. 1975. Polluters' profits and political responses. *American Economic Review* 65:139-147.

Carlton, D. W., and G. C. Loury. 1980. The limitations of Pigouvian taxes as a long-run remedy for externalities. *Quarterly Journal of Economics* 95:559-566.

Coase, R. 1960. The problem of social cost. *Journal of Law and Economics* 3:1-44.

Coase, R. 1981. The Coase theorem and empty core: a comment. *Journal of Law and Economics* 24:183-187.

Cropper, M. L. 1976. Regulating activities with catastrophic environmental effects. *Journal of Environmental Economics and Management* 3:1-15.

Cornes, R., and T. Sandler. 1984. Easy riders, joint production, and public goods. *Economic Journal* 94:580- 598.

Cornes, R., and T. Sandler. 1986. *The Theory of Externalities, Public goods, and Club Goods.* Cambridge University Press, New York.

d'Arge, R. C., W. D. Schulze, and D. S. Brookshire. 1982. Carbon dioxide and intergenerational choice. *American Economic Review* 72:251-256.

Dasgupta, P. S., and G. M. Heal. 1979. *Economic Theory and Exhaustible Resources.* Cambridge University Press, New York.

Dasgupta, P. S., and J. E. Stiglitz. 1977. Tariffs *vs.* quotas as revenue-raising devices under uncertainty. *American Economic Review* 67:975-981.

Dasgupta, P. S., P. Hammond, and E. Maskin. 1980. On imperfect information and optimal pollution control. *Review of Economic Studies* 47:857-860.

Davis, O. A., and A. Whinston. 1962. Externalities, welfare and the theory of games. *Journal of Political Economy* 70:241-262.

Dewees, D. N., and W. A. Sims. 1976. The symmetry of effluent charges and subsidies for pollution control. *Canadian Journal of Economics* 9:323-331.

Farrell, J. 1987. Information and the Coase theorem. *Journal of Economic Perspectives* 1(2):113-129.

Faulhaber, D. R., and W. J. Baumol. 1988. Economists as innovators: Practical products of theoretical research. *Journal of Economic Literature* 26:577-600.

Fishelson, G., and F. Flatters. 1975. The (non-)equivalence of optimal tariffs and quotas under uncertainty. *Journal of International Economics* 5:385-393.

Forster, B. A. 1977. Optimal consumption planning in a polluted environment. *Economic Record* 49:534-545.

Forster, B. A. 1977. Pollution control in a two-sector dynamic general equilibrium model. *Journal of Environmental Economics and Management* 4:305-312.

Heller, W. P., and D. A. Starrett. 1976. On the nature of externalities. Pages 9-21 *in* Lin, S. A. Y., ed. *Theory and Measurement of Economic Externalities.* Academic Press, New York.

Ireland, N. J. 1977. Ideal prices *vs.* quantities. *Review of Economic Studies* 44:183-207.

Just, R. E., and D. Zilberman. 1978. Asymmetry of taxes and subsidies in regulating stochastic mishap. *Quarterly Journal of Economics* 94:139-148.

Kamien, M., and N. Schwartz. 1981. *Dynamic Optimization: The Calculus of Variations and Optimal Control in Economics and Management.* North-Holland, New York.

Kamien, M., N. Schwartz, and F. Dolbear. 1966. Asymmetry between birbes and charges. *Water Resources Research* 2:147-157.

Koenig, E. 1985. Indirect methods for regulating externalities under uncertainty. *Quarterly Journal of Economics* 100:479-493.

Kwerel, E. 1977. To tell the truth: Imperfect information and optimal pollution control. *Review of Economic Studies* 44:595-601.

Laffont, J. J. 1977. More on prices *vs.* quantities. *Review of Economic Studies*

44:183-207.

Lapan, H., and E. K. Choi. 1987. Tariffs *versus* quotas under uncertainty: Restricting imports and the role of preference. Unpublished manuscript. Department of Economics, Iowa State University.

Lerner, A. P. 1971. The 1971 report of the President's Council of Economic Advisers. *American Economic Review* 61:527-530.

Malcomson, J. M. 1977. Prices vs. quantities: A critical note on the use of approximations. *Review of Economic Studies* 48:203-207.

Marshall, Alfred. 1920. *Principles of economics.* 8th ed. Macmillan Co., Ltd., London.

Meade, J. 1952. External economies and diseconomies in a competitive situation. *Economic Journal* 62:205-217.

Meade, J. 1973. *The theory of Economic Externalities in The Control of Environmental Pollution and Similar Social Cost.* Sijhoff, Leiden, Sweden.

Mendelsohn, R. 1980. An economic analysis of air pollution from coal-fired power plants. *Journal of Environmental Economics and Management* 7:30-41.

Mendelsohn, R. 1986. Regulating heterogenous emissions. *Journal of Environmental Economics and Management* 13:301-312.

Mishan, E. J. 1971. The postwar literature on externalities: an interpretative essay. *Journal of Economic Literature* 9:1-28.

Olson, M., and R. Zeckhauser. 1970. The efficient production of external economies. *American Economic Review* 60:512- 517.

Pelcovits, M. G. 1976. Quotas vs. tariffs. *Journal of International Economics* 6:363-370.

Plott, C. R., and S. Mestelman. 1968. A note on the symmetry between birbes and charges. *Water Resources Research* 4:195-197.

Polinsky, A. M., and S. Shavell. 1982. Pigouvian taxation and administrative costs. *Journal of Public Economics* 19:385- 394.

Porter, R. C. 1974. The long-run asymmetry of subsidies and taxes as anti-pollution policies. *Water Resources Research* 10:415-417.

Repullo, R. 1982. A note on imperfect information and optimal pollution control. *Review of Economic Studies* 49:483-484.

Samuelson, P. A. 1970. The fundamental approximation theorem of portfolio analysis in terms of means, variances and higher moments. *Review of Economic Studies* 37:537-542.

Schulze, W. D., D. S. Brookshire, and T. Sandler. 1981. The social rate of discount for nuclear waste storage: economics or ethics? *Natural Resources Journal* 21:811-832.

Scitovsky, Tibor. 1954. Two concept of external economics. *Journal of Political Economy* 62:70-82.

Segerson, K. 1988. Uncertainty and incentives for non-point pollution control. *Journal of Environmental Economics and Management* 15:87-98.

Shortle, J. S., and J. A. Miranowski. 1987. Intertemporal soil resource use: Is it socially excessive? *Journal of Environmental Economics and Management* 14:99-111.

Sims, W. A. 1981. The short-run asymmetry of pollution subsidies and charges. *Journal of Environmental Economics and Management* 8:395-399.

Tullock, G. 1966. Asymmetry between bribes and charges: a comment. *Water Resources Research* 2:854-855.

Turvey, R. 1963. On divergences between social cost and private cost. *Economica* 30:309-313.

Viner, J. 1953. Cost and supply curves. Pp. 148-172. *In Readings in Price Theory*. The Blakiston Co., Philadelphia, Pa., 1953.

Weitzman, M. L. 1974. Prices *vs.* quantities. *Review of Economic Studies* 45:229-238.

Weitzman, M. L. 1977. Reply to "prices *vs.* quantities: A critical note on the use of approximations" by James M. Malcomson. *Review of Economic Studies*

48:209- 910.

Yohe, G. W. 1978. Toward a general comparison of price controls and quantity controls under uncertainty. *Review of Economic Studies* 45:229-238.

Young, L., and J. E. Anderson. 1980. The optimal policies for restricting trade under uncertainty. *Review of Economic Studies* 46:927-932.

Young, L., and J. E. Anderson. 1982. Risk aversion and optimal trade restrictions. *Review of Economic Studies* 49:291-305.